Name: Key UFID:

Quiz 8B

MAC2313 L23-L25

1. [6 pts] Find the volume of the solid bounded above by the cone $z = \sqrt{x^2 + y^2}$ and below by the ring $4 \leq x^2 + y^2 \leq 16$.

$4 \leq x^2 + y^2 \leq 16$

$4 \leq r^2 \leq 16$

$2 \leq r \leq 4$

$z = \sqrt{x^2 + y^2}$

$z = \sqrt{r^2}$

$z = r$

$z \geq 0$

$0 \leq z \leq r$

$0 \leq \theta \leq 2\pi$ ← cone and ring centered at origin

Option 1:

$$\int_0^{2\pi} \int_2^4 (r)\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{r^3}{3} \Big|_2^4 d\theta$$

$$= \int_0^{2\pi} \frac{64}{3} - \frac{8}{3}\, d\theta$$

$$= \int_0^{2\pi} \frac{56}{3}\, d\theta$$

$$= \frac{56}{3}\theta \Big|_0^{2\pi} = \frac{112\pi}{3}$$

Option 2:

$$\int_0^{2\pi} \int_2^4 \int_0^r 1\, r\, dz\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_2^4 rz \Big|_0^r dr\, d\theta$$

$$= \int_0^{2\pi} \int_2^4 r^2\, dr\, d\theta$$

$$= \int_0^{2\pi} \frac{r^3}{3} \Big|_2^4 d\theta = \int_0^{2\pi} \frac{64}{3} - \frac{8}{3}\, d\theta$$

$$= \int_0^{2\pi} \frac{56}{3}\, d\theta = \frac{56}{3}\theta \Big|_0^{2\pi} = \frac{112\pi}{3}$$

2. [4 pts] Set up a triple integral $\iiint dz\, dy\, dx$ to find the volume of the tetrahedron in the first octant enclosed by the coordinate planes and the plane $6x + y + z = 10$. **Do NOT evaluate the integral.**

$6x + y + z = 10$

$z = 10 - 6x - y$

$0 = 10 - 6x - y$

$y = 10 - 6x$

$0 = 10 - 6x$

$6x = 10$

$x = \frac{5}{3}$

1st octant $\Rightarrow$ $x, y, z \geq 0$

$$\int_0^{5/3} \int_0^{10-6x} \int_0^{10-6x-y} dz\, dy\, dx$$