Key

MAC2313 - Quiz 8C
Spring 2025

1. Find the value of the integral $\int_C (x^2 + y^2 + z^2)ds$, where C is a part of the helix parametrized by $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$, $0 \le t \le 2\pi$.

$r'(t) = \langle -\sin t, \cos t, 1 \rangle \quad |r'(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$

$$\int_C x^2+y^2+z^2 \, ds = \int_0^{2\pi} (\cos^2 t + \sin^2 t + t^2)\sqrt{2}\, dt = \sqrt{2}\int_0^{2\pi} 1 + t^2 \, dt$$

$$= \sqrt{2}\left(t + \frac{t^3}{3}\right)\Big|_0^{2\pi} = \sqrt{2}\left(2\pi + \frac{8\pi^3}{3}\right) = 2\pi\sqrt{2} + \frac{8\pi^3\sqrt{2}}{3}$$

2. Determine if the following vector fields are conservative or not:

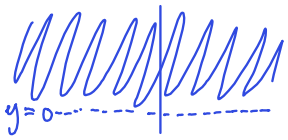
(a) $\mathbf{F}(x, y, z) = \langle xy^2z, x^2yz, z^2 \rangle$ (P, Q, R) $\mathbb{R}^3$, open & simply connected

$\frac{\partial P}{\partial y} \stackrel{?}{=} \frac{\partial Q}{\partial x}$ $\quad$ $\frac{\partial P}{\partial z} \stackrel{?}{=} \frac{\partial R}{\partial x}$ $\quad$ $\frac{\partial Q}{\partial z} \stackrel{?}{=} \frac{\partial R}{\partial y}$

$2xyz = 2xyz$ ✓ $\quad$ $xy^2 \neq 0$ ✗ $\quad$ $x^2y \neq 0$ ✗

$\mathbf{F}$ is not conservative.

(b) $\mathbf{F}(x, y) = \left\langle x\ln(y), \frac{x^2}{2y} \right\rangle$ $\quad \ln(y) \Rightarrow y > 0 \quad \frac{x^2}{2y} \Rightarrow y \neq 0$



open and simply connected

$\frac{\partial P}{\partial y} \stackrel{?}{=} \frac{\partial Q}{\partial x}$

$\frac{x}{y} = \frac{x}{y}$ ✓

since we are on an open & simply connected region, we know $\mathbf{F}$ is conservative.