Name: Key **Section**:

Problem 1 (6 points). Determine whether

$$\mathbf{F} = \langle ye^{xy}, xe^{xy} + 3y^2 \rangle$$

is conservative. If it is, find a function f such that $\nabla f = \mathbf{F}$.

$F(x,y) = \langle P(x,y), Q(x,y) \rangle$

$P = ye^{xy}$ $\quad u = y \quad v = e^{xy}$ $\quad u' = 1 \quad v' = xe^{xy}$

$\frac{dP}{dy} = vu' + uv'$

$= e^{xy} + xye^{xy}$

$Q = xe^{xy} + 3y^2$

$\frac{dQ}{dx} = e^{xy} + xye^{xy}$

Since $\frac{dP}{dy} = \frac{dQ}{dx}$, F is conservative.

To find f:

$\int P(x,y)\,dx = \int ye^{xy}\,dx = e^{xy} + C$

$\int Q(x,y)\,dy = \int xe^{xy} + 3y^2\,dy = e^{xy} + y^3 + C$

$f(x,y) = e^{xy} + y^3 \; (+C)$

(you can check to see that $\nabla f = F$)

Problem 2 (4 points). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F} = \langle -2x, y \rangle$$

, and the curve C is parameterized by

$$\mathbf{r}(t) = \langle \underbrace{t + t^2}_{=x}, \underbrace{t}_{=y} \rangle$$

for $0 \leq t \leq 1$.

Formula:

$\int_a^b F(r) \cdot r'(t)\,dt$

$F(r(t)) = \langle -2(t+t^2), t \rangle$

$r'(t) = \langle 1 + 2t, 1 \rangle$

$F(t) \cdot r'(t)$

$= \langle -2t^2 - 2t, t \rangle \cdot \langle 1 + 2t, 1 \rangle$

$= (-2t^2 - 2t)(1 + 2t) + t$

(simplify)

$\int_0^1 -4t^3 - 6t^2 - t \; dt$

$= -t^4 - 2t^3 - \frac{1}{2}t^2 \Big|_0^1$

$= -1 - 2 - \frac{1}{2} = \frac{-7}{2}$