UF UNIVERSITY *of* FLORIDA
Department of Mathematics

MAC 2313
Final Exam A
Spring 2018

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) Section number

C. Under "special codes" code in the test ID numbers 4, 1.

1 2 3 ● 5 6 7 8 9 0
● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. 1) This test consists of 24 multiple choice questions. The test is counted out of 100 points, and there are 12 bonus points available.

2) The time allowed is 120 minutes.

3) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron to your discussion leader or exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas within one day after the exam.

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ______________________________

Summary of Integration Formulas

- Fundamental Theorem of Calculus

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

- Fundamental Theorem of Line Integrals

$$\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

- Green's Theorem (circulation form)

$$\iint\limits_D \text{curl}\,\vec{F} \cdot \hat{k}\,dA = \oint_C \vec{F} \cdot d\vec{r}$$

- Stokes' Theorem

$$\iint\limits_S \text{curl}\,\vec{F} \cdot \hat{n}\,dS = \oint_C \vec{F} \cdot d\vec{r}$$

- Green's Theorem (flux form)

$$\iint\limits_D \text{div}\,\vec{F}\,dA = \oint_C \vec{F} \cdot \hat{n}\,ds$$

- Divergence Theorem

$$\iiint\limits_E \text{div}\,\vec{F}\,dV = \iint\limits_S \vec{F} \cdot \hat{n}\,dS$$

NOTE: Be sure to bubble the answers to questions 1–24 on your scantron.

$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$

Questions 1 – 20 are worth 5 points each.

$\text{curl}\,\vec{F} = \nabla \times \vec{F} = \det \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$

$= \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle$

1. Let $\vec{F}(x, y, z) = \langle yz^2, x^2z, xy^2 \rangle$. Find curl $\vec{F}$ at $(1, 2, -1)$.

$\text{curl}\,\vec{F} = \det \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz^2 & x^2z & xy^2 \end{vmatrix}$

a. $\langle -3, 8, -3 \rangle$

b. $\langle 3, -8, 3 \rangle$

c. $\langle 3, 8, -3 \rangle$

d. $\langle 3, -8, -3 \rangle$

e. $\langle 3, 8, 3 \rangle$

$= \langle 2xy - x^2, -(y^2 - 2yz), 2xz - z^2 \rangle$

@ $(1,2,-1) = \langle 4-1, -(4+4), -2-1 \rangle = \langle 3, -8, -3 \rangle$

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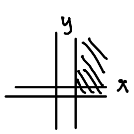
2. Which of the following vector fields are **conservative** on their respective given regions?

$\vec{F} = \overset{\langle P, Q\rangle}{\langle -y, x \rangle}$ on $\mathbb{R}^2$ $\rightarrow$ $\mathbb{R}^2$ simply connected with $\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$ $\Rightarrow$ $\vec{F}$ not conservative

$-1 \neq 1$



$\vec{G} = \left\langle \frac{y}{x}, \ln(x) \right\rangle$ on $\{(x, y) \mid x > 1 \text{ and } y > -1\}$

open, simply connected $\rightarrow$ $\frac{\partial P}{\partial y} = \frac{1}{x}$ $\quad \frac{\partial Q}{\partial x} = \frac{1}{x}$ ✓

$\vec{H} = \langle ye^{xy}, xe^{xy} \rangle$ on $\mathbb{R}^2$

$\frac{\partial P}{\partial y} = e^{xy} + yxe^{xy}$ $\quad \frac{\partial Q}{\partial x} = e^{xy} + xye^{xy}$ ✓

a. $\vec{F}$ and $\vec{H}$ only

b. $\vec{G}$ only

c. $\vec{G}$ and $\vec{H}$ only

d. $\vec{F}$ and $\vec{G}$ only

e. $\vec{F}$, $\vec{G}$, and $\vec{H}$

Recall the theorem that says if $\vec{F} = \langle P, Q \rangle$ is on an open, simply connected region, then $\vec{F}$ is conservative iff $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$.

3. The figure shows a vector field $\vec{F}$ and two curves C_1 and C_2. Which of the following is correct?

it is a "positive" thing for a curve to go "with" the flow of vectors!

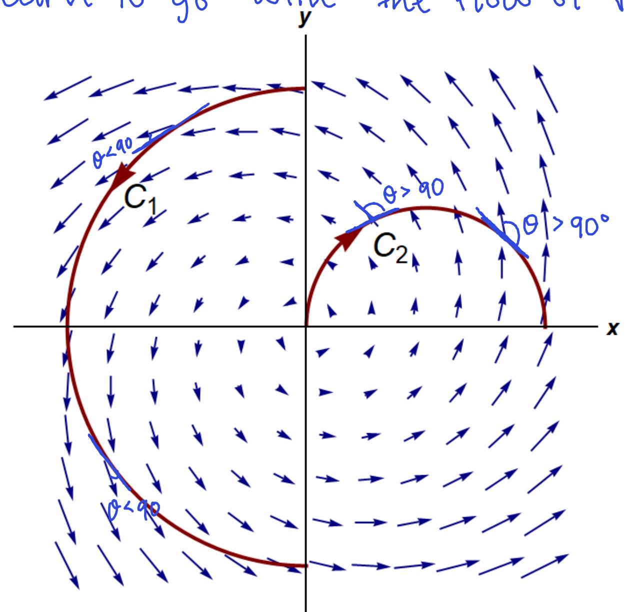
a. $\int_{C_1} \vec{F} \cdot d\vec{r} > \int_{C_2} \vec{F} \cdot d\vec{r} > 0$

b. $\int_{C_1} \vec{F} \cdot d\vec{r} > 0 > \int_{C_2} \vec{F} \cdot d\vec{r}$

c. $0 > \int_{C_2} \vec{F} \cdot d\vec{r} > \int_{C_1} \vec{F} \cdot d\vec{r}$

d. $\int_{C_2} \vec{F} \cdot d\vec{r} > 0 > \int_{C_1} \vec{F} \cdot d\vec{r}$

e. None of the above



if angle between vector and tangent line is <90° then F·dr > 0
if same angle is >90° then F·dr < 0
if same angle is =90° then F·dr = 0

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4. Suppose $F(x, y) = \langle -y, x\rangle$. Let C_1, C_2 and C_3 be the linear paths respectively parameterized by

$$\vec{r}_1(t) = \langle 2t, 2t\rangle, \qquad \vec{r}_2(t) = \langle 2-4t, 2\rangle, \qquad \vec{r}_3(t) = \langle -2+2t, 2-2t\rangle$$

$dr = \langle 2, 2\rangle$ $dr = \langle -4, 0\rangle$ $dr = \langle 2, -2\rangle$

for $0 \le t \le 1$. If $C = C_1 \cup C_2 \cup C_3$, then evaluate $\int_C \vec{F} \cdot d\vec{r}$.

a. -8

b. -2

c. 0

d. 2

e. 8

$C_1: \int_0^1 \langle -y, x\rangle \cdot \langle 2, 2\rangle = \int_0^1 \langle -2t, 2t\rangle \cdot \langle 2, 2\rangle \, dt$

$= \int_0^1 -4t + 4t \, dt = 0$

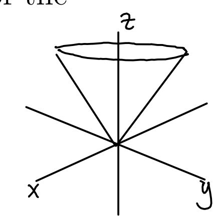
$C_2: \int_0^1 \langle -2, 2-4t\rangle \cdot \langle -4, 0\rangle \, dt = \int_0^1 8 \, dt = 8t\Big|_0^1 = 8$

$C_3: \int_0^1 \langle 2t-2, 2t-2\rangle \cdot \langle 2, -2\rangle \, dt$

$= \int_0^1 2(2t-2) - 2(2t-2) \, dt = 0$

$0 + 8 + 0 = 8$

5. Let S be the part of the cone $z = \sqrt{2x^2 + 2y^2}$ with $0 \le z \le 2$. Then the area of the surface $S =$



$$\iint dS = \iint_R \sqrt{1+\left(\frac{dz}{dx}\right)^2+\left(\frac{dz}{dy}\right)^2}\,dxdy$$

$$= \iint_R \sqrt{1+\left(\frac{dz}{dr}\right)^2+\left(\frac{dz}{d\theta}\right)^2}\,rdrd\theta$$

a. $\int_0^{2\pi}\int_0^1 \sqrt{2}\,dr\,d\theta$

b. $\int_0^{2\pi}\int_0^2 \sqrt{5}\,r\,dr\,d\theta$

c. $\int_0^{2\pi}\int_0^2 \sqrt{3}\,dr\,d\theta$

d. $\int_0^{2\pi}\int_0^{\sqrt{2}} \sqrt{2}\,r\,dr\,d\theta$

e. $\int_0^{2\pi}\int_0^{\sqrt{2}} \sqrt{3}\,r\,dr\,d\theta$

$z = \sqrt{2r^2} = \sqrt{2}\,r$

$\frac{dz}{dr} = \sqrt{2}$ $\qquad \frac{dz}{d\theta} = 0$

$$\iint_R \sqrt{1+(\sqrt{2})^2+(0)^2}\,rdrd\theta = \int_0^{2\pi}\int_0^{\sqrt{2}} \sqrt{3}\,rdrd\theta$$

$z = \sqrt{2}\,r$

$0 = \sqrt{2}\,r$ $\qquad 2 = \sqrt{2}\,r$

$0 = r$ $\qquad \frac{2}{\sqrt{2}} = \sqrt{2} = r$

for r bounds

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6. Suppose $f(x, y, z)$ is a potential function of $\vec{F}(x, y, z) = \langle 2x - z, z, -x + y\rangle$. If $f(1, 1, 0) = 3$, then what is $f(-1, 2, 3)$?

a. 12

b. 6

c. 10

d. 2

e. -2

$\int 2x - z\;dx = x^2 - zx + C$

$\int z\;dy = zy + C$

$\int -x + y\;dz = -zx + yz + C$

$f = x^2 - xz + yz + C$

$f(1,1,0) = 3 = 1 - 0 + 0 + C$

$2 = C$

$f = x^2 - xz + yz + 2$

$f(-1, 2, 3) = (-1)^2 - (-1)(3) + (2)(3) + 2 = 12$

For Questions 7–8: Consider the surface S parameterized by

$$S:\ \vec{r}(u,v) = \langle 3\cos(u), v, \sin(u)\rangle\,,\ 0 \le u \le 2\pi,\ -1 \le v \le 1.$$

7. Identify the surface S.

$x = 3\cos(u)$ $y = v$ $z = \sin(u)$

$\frac{x^2}{9} = \cos^2(u)$ $z^2 = \sin^2(u)$

a. Ellipse

b. Cylinder

c. Cone

d. Paraboloid

e. Ellipsoid

$$\frac{x^2}{9} + z^2 = 1$$

$$F = \frac{x^2}{9} + z^2 - 1$$

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8. If $\hat{n}$ is the unit normal vector (with non-negative z component) to the surface S at $\vec{r}(\pi/4, 0)$, then which of the following vectors is in the same direction as $\hat{n}$?

a. $\langle \sqrt{2}, 0, \sqrt{2}\rangle$

b. $\langle -\sqrt{2}, 0, \sqrt{2}\rangle$

c. $\langle -1, 0, 3\rangle$

d. $\langle 1, 0, 3\rangle$

e. $\langle 1, 0, \sqrt{2}\rangle$

$r(\frac{\pi}{4}, 0) = \langle 3\cos(\frac{\pi}{4}), 0, \sin(\frac{\pi}{4})\rangle$

$\vec{r}(\frac{\pi}{4}, 0) = \langle 3(\frac{\sqrt{2}}{2}), 0, \frac{\sqrt{2}}{2}\rangle$
$= \langle \frac{3\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\rangle$

make the u,v point into an x,y,z point

$$F = \frac{x^2}{9} + z^2 - 1$$

$$\nabla F = \langle \tfrac{2}{9}x, 0, 2z\rangle$$

$$|\nabla F| = \sqrt{\tfrac{4}{81}x^2 + 4z^2}$$

$$@\left(\tfrac{3\sqrt{2}}{2}, 0, \tfrac{\sqrt{2}}{2}\right) = \sqrt{\tfrac{2}{9} + \tfrac{18}{9}} = \frac{\sqrt{20}}{3}$$

$$\boxed{\hat{n} = \frac{\nabla F}{|\nabla F|}} = \frac{3}{\sqrt{20}}\langle \tfrac{2}{9}x, 0, 2z\rangle$$

$$= \left\langle \frac{2}{3\sqrt{20}}x, 0, \frac{6}{\sqrt{20}}z \right\rangle$$

$$\hat{n}\left(\tfrac{3\sqrt{2}}{2}, 0, \tfrac{\sqrt{2}}{2}\right) = \left\langle \frac{2}{3\sqrt{20}}\left(\frac{3\sqrt{2}}{2}\right), 0, \frac{6}{\sqrt{20}}\left(\frac{\sqrt{2}}{2}\right)\right\rangle$$

$$= \left\langle \frac{\sqrt{2}}{\sqrt{20}}, 0, \frac{3\sqrt{2}}{\sqrt{20}} \right\rangle$$

multiply by a scalar of $\frac{\sqrt{20}}{\sqrt{2}}$ to get (d).

P Q R

9. Suppose $\vec{F}(x,y,z) = \langle xy^2, yz^2, zx^2 \rangle$ and let S be the union of the upper hemisphere of S^2 of radius 2 centered at the origin with the disc of radius 2 in the xy-plane centered at the origin such that S is positively oriented. Evaluate $\iint_S \vec{F}(x,y,z) \cdot d\vec{S}$. $= \iiint_E \operatorname{div} \vec{F}\, dV$

$\operatorname{div} \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = y^2 + z^2 + x^2$

a. 0

b. $8\pi^2/3$

c. $16\pi^2/3$

d. $128\pi/5$

e. $64\pi/5$

$\iiint_E x^2+y^2+z^2\, dV = \iiint \rho^2 \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$

$= \int_0^{2\pi}\int_0^{\pi/2}\int_0^2 \rho^4 \sin\phi \, d\rho\, d\phi\, d\theta = \int_0^{2\pi}\int_0^{\pi/2} \frac{1}{5}\rho^5 \sin\phi \Big|_0^2 \, d\phi\, d\theta$

$= \int_0^{2\pi}\int_0^{\pi/2} \frac{32}{5}\sin\phi \, d\phi\, d\theta = \int_0^{2\pi} -\frac{32}{5}\cos\phi \Big|_0^{\pi/2} d\theta = \int_0^{2\pi} \frac{32}{5}\, d\theta$

$= \frac{32}{5}\theta \Big|_0^{2\pi} = \frac{64}{5}\pi$

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10. Which of the following regions is/are simply connected on $\mathbb{R}^2$?

$P = \{(x,y) \mid x^2+y^2 < 1\}$

$Q = \{(x,y) \mid 0 < x^2+y^2 < 1\}$

$R = \{(x,y) \mid 1 < x^2+y^2 < 9 \text{ and } y > 0\}$

a. P only

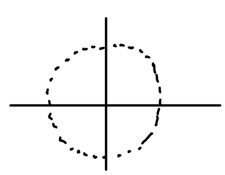
b. Q only

c. R only

d. P and Q

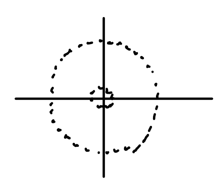
e. P and R

P:



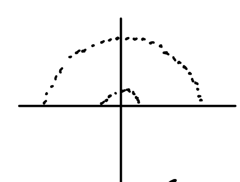
✓

Q:



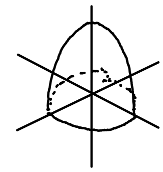
X

R:



✓

11. If S is the part of the paraboloid $z = 4 - x^2 - y^2$ with $z \geq 0$, $\vec{F}(x, y, z) = \langle -y, x, z \rangle$, and $\hat{n}$ is the upward unit normal on S, then $\displaystyle\iint_S (\text{curl}\, \vec{F}) \cdot \hat{n}\, dS =$ $\oint_{\partial S} \vec{F} \cdot d\vec{r}$



$r(t) = \langle 2\cos t, 2\sin t, 0 \rangle, \quad 0 \leq t \leq 2\pi$

$d\vec{r} = \langle -2\sin t, 2\cos t, 0 \rangle$

$\partial S: \; 4 = x^2 + y^2$

use parameters
$x = r\cos\theta$
$y = r\sin\theta$
to get $r(t)$

a. 0

b. 2π

c. 4π

d. 8π

e. 16π

$\vec{F}(r(t))$ $\quad d\vec{r}$

$$\int_0^{2\pi} \langle -2\sin t, 2\cos t, 0 \rangle \cdot \langle -2\sin t, 2\cos t, 0 \rangle \, dt$$

$$\int_0^{2\pi} 4\sin^2 t + 4\cos^2 t \, dt = \int_0^{2\pi} 4\,dt = 4t\Big|_0^{2\pi} = 8\pi$$

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12. Suppose $\vec{F}(x, y)$ is a **conservative** vector field on $\mathbb{R}^2$. Let C_1 be parameterized by $\vec{r}_1(t) = \langle \cos(t), \sin(t) \rangle$ with $0 \leq t \leq \pi$, let C_2 be the line segment from $(0, -2)$ to $(-1, 0)$, and let C_3 be parameterized by $\vec{r}_3(t) = \langle \cos(t), 2\sin(t) \rangle$ with $-\pi/2 \leq t \leq 0$. If $\displaystyle\int_{C_1} \vec{F} \cdot d\vec{r} = 4$ and $\displaystyle\int_{C_2} \vec{F} \cdot d\vec{r} = 3$, then what is $\displaystyle\int_{C_3} \vec{F} \cdot d\vec{r}$?

Hint: Draw the curves.

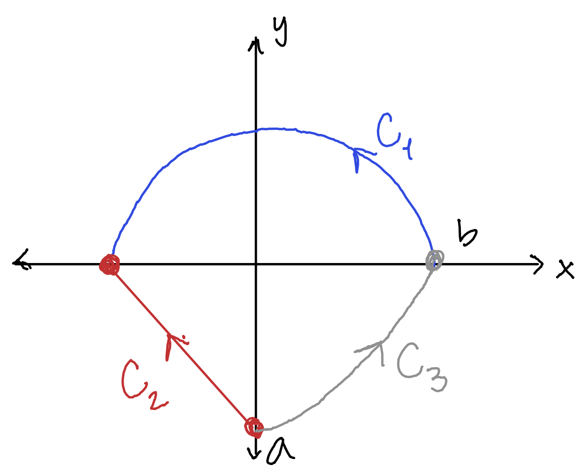
a. 12

b. -7

c. -1

d. 7

e. 0



to go from $a \to b$ we can do C_3 or

$C_2 - C_1 = 3 - 4 = -1$

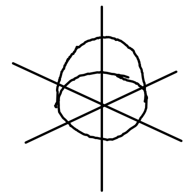
13. Let $E = \{(x, y, z) \mid x^2 + y^2 + z^2 \le 1 \text{ and } z \ge 0\}$, the unit ball with non-negative z. Which of the following describes ∂E (the boundary of E)?

a. $\{(x, y, z) \mid x^2 + y^2 + z^2 = 1,\ z \ge 0\}$

b. $\{(x, y, z) \mid x^2 + y^2 \le 1,\ z = 0\}$

c. $\{(x, y, z) \mid x^2 + y^2 + z^2 = 1,\ z \ge 0\} \cup \{(x, y, z) \mid x^2 + y^2 \le 1,\ z = 0\}$

sphere + disk



d. $\{(x, y, z) \mid x^2 + y^2 = 1,\ z = 0\}$

e. $\{(x, y, z) \mid x^2 + y^2 = 1,\ z = 0\} \cup \{(x, y, z) \mid x^2 + y^2 + z^2 = 1,\ z > 0\}$

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14. Find the work done by the force field $\vec{F} = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$ in moving a particle along the curve $\vec{r}(t) = \langle \cos(t), \cos^2(t), \cos^5(2t) \rangle$, $0 \le t \le \pi/2$.

a. -1

b. -2

c. 0

d. 1

e. 2

$W = \int_C \vec{F} \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r}$

to make our lives easier we use the fundamental theorem of line integrals:

C a smooth curve given by $\vec{r}(t)$, $a \le t \le b$ with ∇f cts. on C.

Then $\int_a^b \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$.

Note F is conservative, and we find its potential function:

$\int yz\,dx = xyz + C \qquad \int xz\,dy = xyz + C \qquad \int xy\,dz = xyz + C$

$f = xyz + C$

$r(0) = \langle 1, 1, 1 \rangle \qquad r(\pi/2) = \langle 0, 0, -1 \rangle$

$W = \int_0^{\pi/2} \nabla f \cdot d\vec{r} = f(0,0,-1) - f(1,1,1) = 0 - 1 = -1$

15. Evaluate $\int_C y\sin(z)\,ds$, where C is the circular helix given by $\vec{r}(t) = \langle \cos(t), \sin(t), t\rangle$, $0 \le t \le 4\pi$.

$x = \cos t \quad y = \sin t \quad z = t$

$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}\,dt$

$\int_0^{4\pi} \sin^2 t \sqrt{2}\,dt = \sqrt{2}\int_0^{4\pi} \sin^2 t\,dt$

$= \sqrt{2}\underbrace{\left(\frac{t}{2} - \frac{\sin(2t)}{4}\right)}\Big|_0^{4\pi} = \sqrt{2}(2\pi - 0) = 2\sqrt{2}\,\pi$

can memorize $\int \sin^2 x\,dx$ or use a trig identity or IBP.

MCQ only. no need to show work

a. π

b. 2π

c. $\sqrt{2}\,\pi$

d. $2\sqrt{2}\,\pi$

e. $4\sqrt{2}\,\pi$

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16. Suppose $\vec{F}(x, y, z)$ is a vector field with continuous second partial derivatives on $\mathbb{R}^3$. Which of the following must be true?

P. $\operatorname{curl} F = \vec{0}$ only true if F is conservative, which we don't know

Q. If $\operatorname{div} F = 0$, then there exists a vector field G such that $F = \operatorname{curl} G$. since $\mathbb{R}^3$ is simply connected, yes

R. $\operatorname{curl}(\operatorname{curl} F) = \vec{0}$ since curl F may not be zero, this one is false

a. Q only

b. R only

c. P and R only

d. P, Q, and R

e. None of them

17. Use Stoke's Theorem to set up an iterated double integral for $\int_C \vec{F} \cdot d\vec{r}$ if

$$\vec{F}(x,y,z) = \langle x+y, -yz, xz \rangle$$

and C is the positive oriented curve of the intersection of the plane $z = x+1$ and the cylinder $x^2 + y^2 = 1$.

$\int_C \vec{F}\cdot d\vec{r} = \iint_S \text{curl}\,\vec{F}\cdot d\vec{S} = \iint \text{curl}\,\vec{F}\cdot\langle -z_x, -z_y, 1\rangle\, dA$

a. $\int_0^{2\pi} \int_0^1 r(r\sin\theta + 1)\, dr\, d\theta$

b. $\int_0^{2\pi} \int_0^1 r(r\cos\theta + 1)\, dr\, d\theta$

c. $\int_0^{2\pi} \int_0^1 r(-r\sin\theta - 1)\, dr\, d\theta$

d. $\int_0^{2\pi} \int_0^1 r^2(-r\cos\theta - 1)\, dr\, d\theta$

e. $\int_0^{2\pi} \int_0^1 r^2\cos\theta\, dr\, d\theta$

$$\text{curl}\,\vec{F} = \det\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y & -yz & xz \end{vmatrix}$$

$= \langle 0+y, -(z-0), 0-1\rangle = \langle y, -z, -1\rangle$

$= \langle y, -x-1, -1\rangle$

want to eliminate z

$z = x+1$

$\frac{\partial z}{\partial x} = 1 \qquad \frac{\partial z}{\partial y} = 0$

$\iint \langle y, -x-1, -1\rangle \cdot \langle -1, 0, 1\rangle\, dA$

$= \iint -y-1\, dA = \int_0^{2\pi}\int_0^1 (-r\sin\theta - 1)\, r\, dr\, d\theta$

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18. The vector field $\vec{F}(x,y)$ is shown below. Determine the sign of $\text{div}\,\vec{F}$ at $(0,0)$ and $(8,8)$.

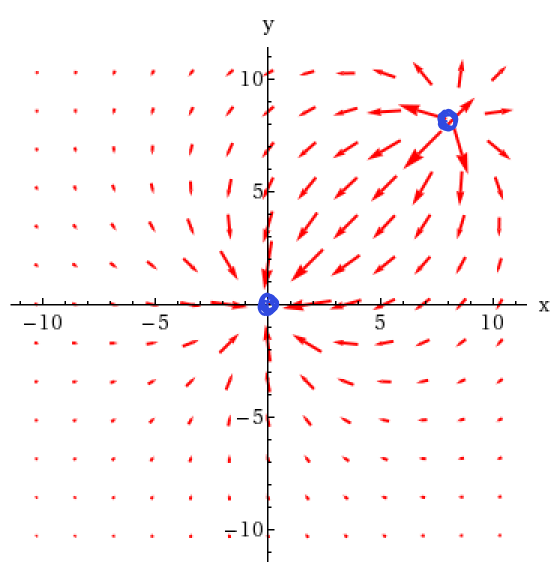
a. Both are positive

b. Both are negative

c. $\text{div}\,\vec{F} > 0$ at $(0,0)$ and $\text{div}\,\vec{F} < 0$ at $(8,8)$

d. $\text{div}\,\vec{F} < 0$ at $(0,0)$ and $\text{div}\,\vec{F} > 0$ at $(8,8)$

e. Cannot determine the sign



19. Let $\vec{F} = \langle xy, x^2 + y^2 \rangle$ be the velocity field of a two-dimensional fluid flow. If D is the semiannular region in the upper half-plane between circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ with a positively oriented boundary, ∂D. Then the flux of $\vec{F}$ across the curve ∂D is $\oint_{\partial D} \vec{F} \cdot \vec{n}\, ds =$

$\oint_{\partial D} \vec{F} \cdot \vec{n}\, dS = \iint_D \nabla \cdot \vec{F}\, dA$

a. $\int_0^{2\pi} \int_1^2 3r^2 \cos\theta \, dr\, d\theta$

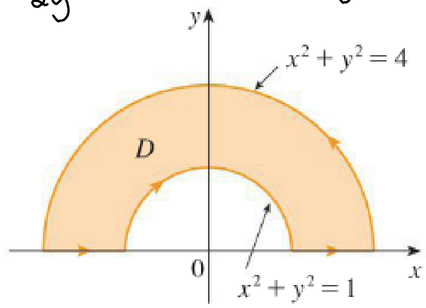
$\nabla \cdot \vec{F} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \rangle \cdot \langle xy, x^2+y^2 \rangle = y + 2y = 3y$

b. $\int_0^{\pi} \int_1^2 3r^2 \sin\theta \, dr\, d\theta$

c. $\int_0^{\pi} \int_0^2 r^2 \cos\theta \, dr\, d\theta$

d. $\int_0^{\pi} \int_1^2 r^2 \sin\theta \, dr\, d\theta$

e. $\int_0^{2\pi} \int_1^2 3r \cos\theta \, dr\, d\theta$



$\iint_D 3y\, dA = \int_0^{\pi} \int_1^2 3r\sin\theta\, r\, dr\, d\theta$

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20. Which of the following is correct?

⊥ to xy-plane, so flux ≠ 0

a. If $\vec{F}(x, y, z) = \langle 1, 0, 0 \rangle$, then the flux of $\vec{F}$ across the yz-plane is zero.

b. If S is a surface parameterized by $\vec{r}(u, v)$, then the vector $\vec{r}_u \times \vec{r}_v$ lies in the tangent plane of S at a given point. no b/c thats a normal vector

c. The vector field $\vec{F}(x, y, z) = \langle x, x, x \rangle$ is independent of path. curl $\vec{F} \neq 0 \Rightarrow$ not path-independent

d. There is a vector field $\vec{F}$ on $\mathbb{R}^3$ such that curl $\vec{F} = \langle 2x, -y, -z \rangle$. div$(\langle 2x, -y, -z \rangle)$

$= 2 - 1 - 1 = 0$

so $\vec{F}$ exists.

e. All of the above are correct.

Bonus Questions 21 − 24 are worth 3 points each.

21. Let $\vec{u} = \langle 3, 6, -9\rangle$ and $\vec{v} = \langle 1, 1, -1\rangle$. If $\vec{u} = \vec{v}_{//} + \vec{v}_{\perp}$, where $\vec{v}_{//}$ is parallel to $\vec{v}$ and $\vec{v}_{\perp}$ is perpendicular to $\vec{v}$, and $\vec{v}_{\perp} = \langle a, b, c\rangle$, then what is a?

a. -3

b. 0

c. 6

d. -6

e. 3

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22. Consider the surface $4x^2 + y^2 - z = 0$. Which of the following is/are correct?

P. The graph of the surface is a paraboloid.

Q. The level curves are hyperbolas.

R. The vertical trace in the the yz-plane is a parabola.

a. P only

b. Q only

c. P and R only

d. Q and R only

e. P, Q, and R

23. Let E be the solid region bounded by the paraboloid $z = 2x^2 + 2y^2$ and the plane $z = 10$. Which of the following integrals represents the volume of E?

a. $\int_0^{2\pi} \int_0^{5} (10r - 2r^3)\, dr\, d\theta$

b. $\int_0^{2\pi} \int_0^{\sqrt{5}} (10r - 2r^3)\, dr\, d\theta$

c. $\int_0^{2\pi} \int_0^{10} (2r^3 - 10r)\, dr\, d\theta$

d. $\int_0^{2\pi} \int_0^{\sqrt{5}} (2r^2 - 10)\, dr\, d\theta$

e. $\int_0^{2\pi} \int_0^{5} (10 - 2r^2)\, dr\, d\theta$

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24. Find the absolute maximum value of $f(x, y) = 1 + xy^2$ on the set $D = \{(x, y) \mid x^2 + y^2 \leq 3,\ x \geq 0,\ y \geq 0\}$.

a. 1 b. 2 c. 3 d. 4 e. 5