UF UNIVERSITY of FLORIDA
*Department of Mathematics*

**CALCULUS III**
**SPRING 2025**
**Version A**

# FINAL EXAM

**A.** Sign your bubble sheet on the back at the bottom in ink.

**B.** In pencil, write and encode in the spaces indicated:

**1)** Name (last name, first initial, middle initial)

**2)** UF ID number

**3)** **SKIP** Section number

**C.** Under "special codes" code in the test ID numbers 4, 1.

1 2 3 ● 5 6 7 8 9 0
● 2 3 4 5 6 7 8 9 0

**D.** At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

**E.** **1)** This test consists of 18 multiple choice questions

**2)** The time allowed is 120 minutes.

**3)** You may write on the test.

**4)** Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

**F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.**

**G.** When you are finished:

**1)** Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

**2)** You must turn in your scantron and tearoff sheets to your discussion leader or exam proctor. Be prepared to show your picture I.D. with a legible signature.

**3)** The answers will be posted in Canvas within one day after the exam.

**Summary of Integration Formulas**

• Fundamental Theorem of Calculus

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

• Fundamental Theorem of Line Integrals

$$\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

• Green's Theorem (circulation form)

$$\iint_D \operatorname{curl} \vec{F} \cdot \hat{k}\,dA = \oint_C \vec{F} \cdot d\vec{r}$$

• Stokes' Theorem

$$\iint_S \operatorname{curl} \vec{F} \cdot \hat{n}\,dS = \oint_C \vec{F} \cdot d\vec{r}$$

• Green's Theorem (flux form)

$$\iint_D \operatorname{div} \vec{F}\,dA = \oint_C \vec{F} \cdot \hat{n}\,ds$$

• Divergence Theorem

$$\iiint_E \operatorname{div} \vec{F}\,dV = \oiint_S \vec{F} \cdot \hat{n}\,dS$$

**Questions 1 – 18 are worth 6 points each.**

1. Evaluate $\int_C xy + 2z\, ds$ where $C$ is the line from $P(1,0,0)$ to $Q(0,1,1)$.

   (a) $\frac{\sqrt{3}}{6}$

   (b) $\frac{5\sqrt{3}}{6}$

   (c) $\frac{7\sqrt{3}}{6}$

   (d) $\frac{\sqrt{5}}{6}$

   (e) $7\frac{\sqrt{5}}{6}$

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2. Evaluate the line integral of $\vec{F} = \langle y-x, x\rangle$ on the path of the quarter circle from $P(0,1)$ to $Q(1,0)$.

   (a) $0$

   (b) $-\frac{5}{2}$

   (c) $-\frac{1}{2}$

   (d) $-\frac{3}{2}$

   (e) $-\frac{15}{2}$

3. Let $C$ be the unit circle with counterclockwise orientation. Find the circulation on $C$ of the rotation flow field $\vec{F} = \langle -y, x \rangle$.

   (a) $0$

   (b) $\pi$

   (c) $2\pi$

   (d) $3\pi$

   (e) $4\pi$

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4. Find a potential function for the vector field $\vec{F} = \langle 2xy - z^2, x^2 + 2z, 2y - 2xz \rangle$.

   (a) $f(x, y, z) = x^2y - xz^2 + 2yz$

   (b) $f(x, y, z) = x^2$

   (c) $f(x, y, z) = x^2y - xz^2$

   (d) $f(x, y, z) = -2xz + 2y$

   (e) $\vec{F}$ is not conservative

5. Find the work done by the vector field $\vec{F} = \langle yz, xz, xy \rangle$ along any smooth curve joining $P(-1, 3, 9)$ to $Q(1, 6, -4)$.

   (a) 0

   (b) 1

   (c) 2

   (d) 3

   (e) 4

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6. Compute the line integral $\oint_C x^2 - y^2 \, dx + x^3 + y^2 \, dy$ where $C$ is the boundary of the region enclosed by $y = x^2$ and $y = x$ .

   (a) $\dfrac{9}{13}$

   (b) 9

   (c) $\dfrac{9}{35}$

   (d) 11

   (e) 13

7. Find the surface area of the cylinder $x^2 + y^2 = 16$ between the planes $z = 0$ and $z = 16 - 2x$.

   (a) $24\pi$

   (b) $48\pi$

   (c) $64\pi$

   (d) $128\pi$

   (e) $264\pi$

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8. Consider the vector field $\vec{F} = \langle 0, 0, -1 \rangle$. Find the flux in the downward (negative $z$) direction across the surface $z = 4 - 2x - y$ in the first octant.

   (a) 1

   (b) 2

   (c) 3

   (d) 4

   (e) 5

9. Use Stokes' Theorem to evaluate the line integral $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = \langle 2y, -z, x \rangle$ and $C$ is the circle $x^2 + y^2 = 12$ with counterclockwise orientation in the plane $z = 0$.

   (a) $-6\pi$

   (b) $-12\pi$

   (c) $-24\pi$

   (d) $-48\pi$

   (e) $-96\pi$

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10. Compute the tangent plane to the surface parametrized by $\vec{r} = u\hat{\imath} + uv\hat{\jmath} + (u+v)\hat{k}$ at the point (1,2,3).

   (a) $3x + 2y + z = 10$

   (b) $x - y + z = 2$

   (c) $x + 2y + 3z = 14$

   (d) $\langle x, y, z \rangle = \langle 1 + u, 2 + uv, 3 + u + v \rangle$

   (e) $\dfrac{x-1}{1} = \dfrac{y-2}{2} = \dfrac{z-3}{3}$

11. Calculate $\iint_S \vec{F} \cdot d\vec{S}$ where $F = \langle 2x^3 + y^3, y^3 + z^3, 3y^2 z \rangle$ and $S$ is the surface of the solid bounded by the paraboloid $z = 1 - x^2 - y^2$ and the $xy$-plane.

   (a) $\pi$

   (b) $2\pi$

   (c) $3\pi$

   (d) $4\pi$

   (e) $5\pi$

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12. Let $\vec{F} = \langle yz + 1, xz + 1, xy + 1 \rangle$. Which of the following statements must be correct?

   P. $\vec{F}$ is conservative.

   Q. $\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_2} \vec{F} \cdot d\vec{r}$ for any two smooth curves $C_1$ and $C_2$.

   R. The circulation of $\vec{F}$ along a smooth curve is zero.

   (a) P and Q only

   (b) P and R only

   (c) Q and R only

   (d) P, Q, and R

   (e) P only

13. Let $\vec{F} = \langle 2x^3 + y^3, 2y^3 + z^3, 2z^3 + x^3 \rangle$ and let $S$ be the sphere centered at $(0, 0, 2)$ with radius 2. By the Divergence Theorem, the flux of $\vec{F}$ across the surface $S$ is

(a) $\displaystyle\int_0^{2\pi}\int_0^{\pi/2}\int_0^{4\cos\phi} 6\rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$

(b) $\displaystyle\int_0^{2\pi}\int_0^{\pi/2}\int_0^{2\cos\phi} 6\rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$

(c) $\displaystyle\int_0^{2\pi}\int_0^{\pi/2}\int_0^{4\cos\phi} 6\rho^2 \, d\rho \, d\phi \, d\theta$

(d) $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{2\cos\phi} 6\rho^2 \, d\rho \, d\phi \, d\theta$

(e) $\displaystyle\int_0^{2\pi}\int_0^{\pi}\int_0^{2} 6\rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$

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14. Let $S$ be the unit sphere centered at the origin. Which of the following vector fields whose flux integral across $S$ is equal to 0?

$\vec{F} = \langle x, y, z \rangle$

$\vec{G} = \langle e^{yz}, -2y, 2z \rangle$

$\vec{H} = \langle x - 2y, y - 2z, z - 2x \rangle$

(a) $\vec{F}$ only

(b) $\vec{G}$ only

(c) $\vec{H}$ only

(d) $\vec{F}$, $\vec{G}$, and $\vec{H}$

(e) none of them

15. Let $S$ be the part of the sphere of radius 1 centered at the origin and lying in the first octant. The surface can be represented parametrically by

(a) $\vec{r}(\theta, \phi) = \langle \cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi \rangle$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$

(b) $\vec{r}(\theta, \phi) = \langle \cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi \rangle$, $0 \le \theta \le \pi$, $0 \le \phi \le \pi$

(c) $\vec{r}(\theta, \phi) = \langle \cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi \rangle$, $0 \le \theta \le \pi$, $0 \le \phi \le \pi/2$

(d) $\vec{r}(\theta, \phi) = \langle \cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi \rangle$, $0 \le \theta \le \pi/2$, $0 \le \phi \le \pi/2$

(e) $\vec{r}(\theta, \phi) = \langle \cos\theta \sin\phi, \sin\theta \sin\phi, \cos\phi \rangle$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi/2$

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16. Let $C$ be given by $\vec{r}(t) = \langle t. - t^2, -2 \rangle$, $0 \le t \le 2$. Compute $\displaystyle\int_C y\,dx + 2z\,dy + (x+z)\,dz$.

(a) $0$

(b) $\dfrac{22}{3}$

(c) $\dfrac{32}{3}$

(d) $\dfrac{64}{3}$

(e) $\dfrac{40}{3}$

17. Let $\vec{F} = \langle z - 2y, z + 2x, e^{-xy} \rangle$ and $S$ be the part of the paraboloid $z = 9 - x^2 - y^2$ with $z \geq 0$ and oriented so that $z$ component of the normal vector is positive. Calculate

$$\iint_S \nabla \times \vec{F} \cdot \vec{n}\, dS =$$

a. $9\pi$

b. $18\pi$

c. $36\pi$

d. $48\pi$

e. $0$

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18. You attend the University of

(a) Florida

(b) Missouri

(c) Arkansas

(d) Kansas