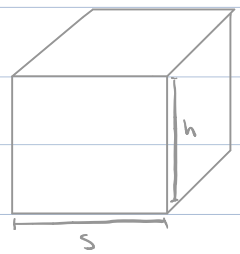
XRONOS HW 21 #5: IF 1200 SQUARE CENTIMETERS OF MATERIAL IS AVAILABLE TO MAKE A BOX WITH A SQUARE BASE AND AN OPEN TOP, FIND THE LARGEST POSSIBLE VOLUME OF THE BOX.



GIVEN: SURFACE AREA $= 1200 cm^2$

SURFACE AREA = (AREA OF BASE) + 4(AREA OF ONE SIDE)

$$1200 = (s \cdot s) + 4(s \cdot h)$$

$$1200 = s^2 + 4sh$$

$$1200 - s^2 = 4sh$$ SOLVE FOR h

$$\frac{1200 - s^2}{4s} = h$$

$V = s \cdot s \cdot h$

$$V = s^2 h \Rightarrow V(s) = s^2\left(\frac{1200 - s^2}{4s}\right) = \frac{1200 s^2}{4s} - \frac{s^4}{4s} = 300s - \frac{1}{4}s^3$$

$$\Rightarrow V(s) = 300s - \frac{1}{4}s^3$$

NOTE: VOLUME CANNOT BE NEGATIVE, SO $300s - \frac{1}{4}s^3 \geq 0$. SOLVING FOR s, WE GET $0 \leq s \leq 20\sqrt{3}$, SO OUR DOMAIN IS $[0, 20\sqrt{3}]$

AT THE END POINTS, $V(0) = V(20\sqrt{3}) = 0$, SO MAX IS ACHIEVED WHEN $V'(s) = 0$.

$V'(s) = 300 - \frac{3}{4}s^2$, SET $V'(s) = 0$: $300 - \frac{3}{4}s^2 = 0$

$$300 = \frac{3}{4}s^2 \Rightarrow 1200 = 3s^2 \Rightarrow 400 = s^2$$

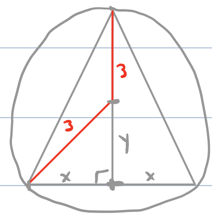
$$\Rightarrow s = \pm 20$$

SINCE THE DOMAIN IS $[0, 20\sqrt{3}]$, WE MUST HAVE $s = 20$.

SO, THE LARGEST POSSIBLE VOLUME IS $V(20) = 300(20) - \frac{1}{4}(20^3) = \boxed{4000 cm^3}$

XRONOS HW 21 #6: FIND THE LARGEST AREA OF AN ISOSCELES TRIANGLE INSCRIBED IN A CIRCLE OF RADIUS 3.

AREA OF TRIANGLE: $A = \overbrace{\frac{1}{\cancel{2}}}^{\frac{1}{2}} \overbrace{(\cancel{2}x)}^{(BASE)} \overbrace{(y+3)}^{(HEIGHT)} = x(y+3)$



↓ PYTHAGOREAN THEOREM

↓ IS POSITIVE B/C IT IS A LENGTH

$3^2 = x^2 + y^2 \Rightarrow 9 - y^2 = x^2 \Rightarrow x = \sqrt{9-y^2}$

$\Rightarrow A(y) = (\sqrt{9-y^2})(y+3) = y\sqrt{9-y^2} + 3\sqrt{9-y^2}$

$A(y) = y(9-y^2)^{1/2} + 3(9-y^2)^{1/2}$

NOTE: THE DOMAIN FOR $A(y)$ IS $[-3,3]$

$A'(y) = (9-y^2)^{1/2} + y(\frac{1}{\cancel{2}})(9-y^2)^{-1/2}(-\cancel{2}y) + 3(\frac{1}{\cancel{2}})(9-y^2)^{-1/2}(-\cancel{2}y)$

$A'(y) = \sqrt{9-y^2} - \frac{y^2}{\sqrt{9-y^2}} - \frac{3y}{\sqrt{9-y^2}} = \frac{9-y^2}{\sqrt{9-y^2}} - \frac{y^2}{\sqrt{9-y^2}} - \frac{3y}{\sqrt{9-y^2}}$

$A'(y) = \frac{9-2y^2-3y}{\sqrt{9-y^2}}$

$A'(y) = 0 \Rightarrow 9 - 2y^2 - 3y = 0 \Rightarrow 2y^2 + 3y - 9 = 0$

$\Rightarrow (2y-3)(y+3) = 0$

$\Rightarrow y = 3/2$ OR $y = -3$

AT $y = -3$, $A(y) = 0$, SO THE MAXIMUM AREA IS ACHIEVED FOR $y = 3/2$.

RECALL, $x = \sqrt{9-y^2} \Rightarrow x = \sqrt{9-(3/2)^2} \Rightarrow x = \sqrt{9 - 9/4} = \sqrt{36/4 - 9/4} = \sqrt{27/4}$

$= \sqrt{27}/2 = 3\sqrt{3}/2$

$\Rightarrow$ MAXIMUM AREA IS ACHIEVED AT $y = 3/2$, $x = 3\sqrt{3}/2$

$\Rightarrow A = x(y+3) \Rightarrow A = \frac{3\sqrt{3}}{2}\left(\frac{3}{2}+3\right) = \frac{3\sqrt{3}}{2}\left(\frac{3}{2}+\frac{6}{2}\right) = \frac{3\sqrt{3}}{2}\left(\frac{9}{2}\right) = \frac{27\sqrt{3}}{4}$

MAXIMUM AREA: $\boxed{\frac{27\sqrt{3}}{4} \text{ UNITS}^2}$

XRONOS HW 21 #2: FIND THE POINT ON $y^2=2x$ THAT IS CLOSEST TO THE POINT $(1,4)$.

SINCE $y^2=2x$, $x=\frac{1}{2}y^2$.

NOW, THE DISTANCE FROM (x,y) TO $(1,4)$ IS: $D=\sqrt{(x-1)^2+(y-4)^2}$

SUBSTITUTE $x=\frac{1}{2}y^2$ FOR x SO THAT D IS A FUNCTION OF y:

$$D(y)=\sqrt{\left(\tfrac{1}{2}y^2-1\right)^2+(y-4)^2}$$

SIMPLIFY

$$D(y)=\left(\left(\tfrac{1}{4}y^4-y^2+1+y^2-8y-16\right)\right)^{1/2}$$

SIMPLIFY

$$D(y)=\left(\tfrac{1}{4}y^4-8y+17\right)^{1/2}$$

TAKE DERIVATIVE AND FIND y SUCH THAT $D'(y)=0$ TO FIND CRITICAL POINTS:

$$D'(y)=\frac{1}{2}\left(\frac{1}{4}y^4-8y+17\right)^{-1/2}\left(y^3-8\right)$$ * CHAIN RULE *

$$D'(y)=\frac{y^3-8}{2\sqrt{\frac{1}{4}y^4-8y+17}}$$

$$D'(y)=0 \Rightarrow y^3-8=0$$

$$y^3=8 \Rightarrow y=2$$

RECALL: $x=\frac{1}{2}y^2$, SO FOR $y=2$, $x=\frac{1}{2}(2^2)=\frac{1}{2}(4)=2$

$\Rightarrow$ THE POINT $(2,2)$ IS CLOSEST TO $(1,4)$

XRONOS HW 21 #4: FIND THE POSITIVE NUMBERS WHOSE PRODUCT IS 100 AND WHOSE SUM IS THE SMALLEST POSSIBLE.

WANT: x,y SUCH THAT $xy=100$ AND $S=x+y$ IS AS SMALL AS POSSIBLE.

x AND y ARE POSITIVE, SO $x,y>0$ (ie $y,x\neq 0$)

$xy=100 \Rightarrow y=\frac{100}{x}$

WRITE S AS A FUNCTION OF X

$S=x+y \Rightarrow S(x)=x+\frac{100}{x}=x+100x^{-1}$

$S'(x)=1+(-100)x^{-2} \Rightarrow S'(x)=1-\frac{100}{x^2}$

$S'(x)=0 \Rightarrow 1-\frac{100}{x^2}=0 \Rightarrow 1=\frac{100}{x^2} \Rightarrow x^2=100 \Rightarrow x=\pm 10$

WE ARE LOOKING FOR POSITIVE NUMBERS, SO TAKE $\boxed{x=10}$

$x=10$, SUBSTITUTE INTO $y=\frac{100}{x} \Rightarrow y=\frac{100}{10} \Rightarrow \boxed{y=10}$

<u>XRONOS HW 21 #2:</u> FIND THE POINT ON $y^2=2x$ THAT IS CLOSEST TO THE POINT $(1,4)$.

SINCE $y^2=2x$, $x=\frac{1}{2}y^2$.

NOW, THE DISTANCE FROM (x,y) TO $(1,4)$ IS: $D=\sqrt{(x-1)^2+(y-4)^2}$

SUBSTITUTE $x=\frac{1}{2}y^2$ FOR x SO THAT D IS A FUNCTION OF y:

$D(y)=\sqrt{\left(\frac{1}{2}y^2-1\right)^2+(y-4)^2}$

SIMPLIFY

$D(y)=\left(\left(\frac{1}{4}y^4-y^2+1+y^2-8y-16\right)\right)^{1/2}$ SIMPLIFY

$D(y)=\left(\frac{1}{4}y^4-8y+17\right)^{1/2}$

TAKE DERIVATIVE AND FIND y SUCH THAT $D'(y)=0$ TO FIND CRITICAL POINTS:

$D'(y)=\frac{1}{2}\left(\frac{1}{4}y^4-8y+17\right)^{-1/2}\left(y^3-8\right)$ * CHAIN RULE *

$$D'(y) = \frac{y^3 - 8}{2\sqrt{\frac{1}{4}y^4 - 8y + 17}}$$

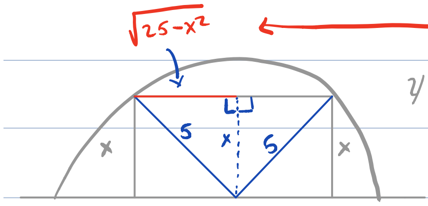
$D'(y) = 0 \Rightarrow y^3 - 8 = 0$

$y^3 = 8 \Rightarrow y = 2$

RECALL: $x = \frac{1}{2}y^2$, SO FOR $y = 2$, $x = \frac{1}{2}(2^2) = \frac{1}{2}(4) = 2$

$\Rightarrow$ THE POINT $(2,2)$ IS CLOSEST TO $(1,4)$

XRONOS HW 21 #3: FIND THE AREA OF THE LARGEST RECTANGLE THAT CAN BE INSCRIBED IN A SEMICIRCLE OF RADIUS 5.



USING PYTHAGOREAN THEOREM

$2/\frac{\sqrt{50}}{4} \cdot \frac{\sqrt{50}}{\cancel{2}} = \frac{50}{4} =$

NOW, THE AREA OF THE RECTANGLE IS $A(x) = x\left(2\sqrt{25 - x^2}\right)$

TAKE DERIVATIVE OF $A(x)$ AND FIND x S.T. $A'(x) = 0$

WRITE $A(x) = 2x(25 - x^2)^{1/2}$

$A'(x) = 2(25 - x^2)^{1/2} + \cancel{2}x\left(\frac{1}{\cancel{2}}(25 - x^2)^{-1/2}(-2x)\right)$

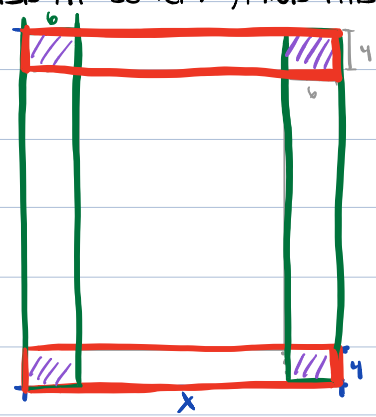
$$A'(x) = 2\sqrt{25 - x^2} - \frac{2x^2}{\sqrt{25 - x^2}}$$

$$A'(x) = \frac{2(25 - x^2)}{\sqrt{25 - x^2}} - \frac{2x^2}{\sqrt{25 - x^2}} \Rightarrow A'(x) = \frac{50 - 2x^2 - 2x^2}{\sqrt{25 - x^2}} \Rightarrow A'(x) = \frac{50 - 4x^2}{\sqrt{25 - x^2}}$$

$A'(x) = 0 \Rightarrow 50 - 4x^2 = 0 \Rightarrow 50 = 4x^2 \Rightarrow x^2 = \frac{50}{4} = \frac{25}{2} \Rightarrow x = \frac{5}{\sqrt{2}}$

Now, $A(x)=2\left(\frac{5}{\sqrt{2}}\right)\left(25-\left(\frac{5}{\sqrt{2}}\right)^2\right)^{1/2}=\frac{10}{\sqrt{2}}\left(\frac{5}{\sqrt{2}}\right)=\frac{50}{2}=\boxed{25 \text{ SQUARE UNITS}}$

XRONOS HW 21 #7: THE TOP AND BOTTOM MARGINS OF A POSTER ARE 6CM EACH, AND THE SIDE MARGINS ARE 4CM EACH. IF THE AREA OF THE PRINTED MATERIAL ON THE POSTER IS FIXED AT $384 cm^2$, FIND THE DIMENSIONS OF THE POSTER OF SMALLEST AREA.



$A = xy$ ↙ AREA OF POSTER

AREA OF PRINTED = 384

$\Rightarrow A = 384 + 2(x\cdot 4) + 2(y\cdot 6) - 4(6\cdot 4)$

$\Rightarrow A = 384 + 8x + 12y - 96 = 288 + 8x + 12y$

(B/C $A=xy$)

$xy = 288 + 8x + 12y$

$xy - 8x = 288 + 12y$

$x(y-8) = 288 + 12y$

$x = \frac{288+12y}{y-8}$

(PLUG IN X TO $A=xy$)

$\Rightarrow A(y) = y\left(\frac{288+12y}{y-8}\right) = \frac{288y+12y^2}{y-8}$

$A'(y) = \frac{(288+24y)(y-8)-(288y+12y^2)(1)}{(y-8)^2}$ ↙ QUOTIENT RULE

$A'(y) = \frac{288y - 2304 + 24y^2 - 192y - 288y - 12y^2}{(y-8)^2}$

$A'(y) = \frac{12y^2 - 192y - 2304}{(y-8)^2}$

$A'(y) = 0 \Rightarrow 12y^2 - 192y - 2304 = 0$

$\Rightarrow 12(y^2 - 16y - 192) = 0$

$\Rightarrow 12(y+8)(y-24)=0$

$\Rightarrow y=-8$ OR $y=24$

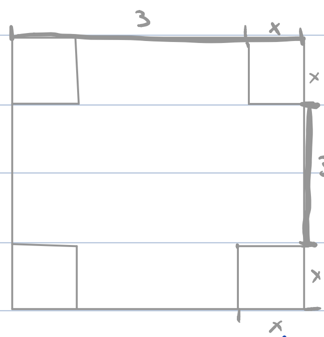
↑SINCE WE ARE TALKING ABOUT HEIGHT, y CANNOT BE NEGATIVE,

SO MAXIMUM HEIGHT IS 24

NOW, $x=\frac{288+12y}{y-8} \Rightarrow x=\frac{288+12(24)}{24-8}=36$

$\Rightarrow$ THE DIMENSIONS OF THE POSTER OF SMALLEST AREA: $\boxed{24cm \times 36cm}$

<u>XRONOS HW 21 #8</u>: A BOX WITH AN OPEN TOP IS TO BE CONSTRUCTED FROM A SQUARE PIECE OF CARDBOARD THAT IS 3 FEET ON EACH SIDE, BY CUTTING OUT A SQUARE FROM EACH OF THE 4 CORNERS AND BENDING UP THE SIDES. WHAT IS THE LARGEST VOLUME THAT SUCH A BOX CAN HAVE?



$\Rightarrow V=(3-2x)^2x$

$V=(9-12x+4x^2)x$

$V=9x-12x^2+4x^3$

$\Rightarrow$ FIND $V'(x)$ AND SET IT EQUAL TO 0:

$V(x)=4x^3-12x^2+9x$

$V'(x)=12x^2-24x+9$

$V'(x)=0 \Rightarrow 12x^2-24x+9=0 \Rightarrow 3(4x^2-8x+3)=0 \Rightarrow 4x^2-8x+3=0$

$\Rightarrow (2x-3)(2x-1)=0$

$\Rightarrow x=\frac{3}{2}, \frac{1}{2}$

$*$ IF $x = \frac{3}{2}$, THEN $3-2x = 3-2\left(\frac{3}{2}\right) = 3-3 = 0$, SO THE LENGTH OF THE BOX WOULD BE 0, SO $x \neq \frac{3}{2}$

$\Rightarrow x = \frac{1}{2}$, SO $V(x) = V\left(\frac{1}{2}\right) = \left(3-2\left(\frac{1}{2}\right)\right)^2\left(\frac{1}{2}\right) = (3-1)^2\left(\frac{1}{2}\right) = 2^2\left(\frac{1}{2}\right)$

$= 4\left(\frac{1}{2}\right) = 2$

$\Rightarrow$ MAXIMUM VOLUME IS $\boxed{\text{2 CUBIC FEET}}$

XRONOS HW 22 #10:

DETERMINE $f(x)$, GIVEN $f''(x) = -\cos(x) + \sin(x)$, $f(0)=1$, $f(\pi)=0$

ANTIDERIVATIVE OF $f''(x)$: $f'(x) = -\sin(x) - \cos(x) + C$

ANTIDERIVATIVE OF $f'(x)$: $f(x) = \cos(x) - \sin(x) + Cx + K$

$f(0)=1 \Rightarrow \cos(0) - \sin(0) + C(0) + k = 1$

$1 - 0 + 0 + k = 1$

$1 + k = 1$

$k = 0$

$f(x) = \cos(x) - \sin(x) + cx$

$f(\pi) = 0 \Rightarrow \cos(\pi) - \sin(\pi) + c(\pi) = 0$

$-1 - 0 + c\pi = 0$

$c\pi = 1$

$c = \frac{1}{\pi}$

$\Rightarrow \boxed{f(x) = \cos(x) - \sin(x) + \frac{1}{\pi}x}$

XRONOS HW 22 #11:

IF THE ACCELERATION OF A PARTICLE ON A STRAIGHT LINE IS GIVEN BY $a(t) = 2t+1$

WITH $s(0) = 3$ AND $v(0) = -2$, WHAT IS THE POSITION FUNCTION $s(t)$?

ANTIDERIVATIVE OF $a(t)$: $v(t) = \frac{2t^{1+1}}{1+1} + 1t + C$

$v(t) = \frac{2t^2}{2} + t + C \Rightarrow v(t) = t^2 + t + C$

$v(0) = -2 \Rightarrow 0^2 + 0 + C = -2 \Rightarrow C = -2 \Rightarrow v(t) = t^2 + t - 2$

ANTIDERIVATIVE OF $v(t)$: $s(t) = \frac{t^{2+1}}{2+1} + \frac{t^{1+1}}{1+1} - 2t + C$

$$s(t) = \frac{t^3}{3} + \frac{t^2}{2} - 2t + C$$

$s(0)=3 \Rightarrow \frac{0^3}{3} + \frac{0^2}{2} - 2(0) + C = 3 \Rightarrow C = 3$

$$\boxed{s(t) = \frac{t^3}{3} + \frac{t^2}{2} - 2t + 3}$$

XRONOS HW22#5:

FIND THE ANTIDERIVATIVE: $f(x) = x^2(\sqrt[3]{x}) - 4\cot(x)\csc(x)$

$f(x) = x^2(x^{1/3}) - 4\cot(x)\csc(x)$ $\quad 2 + \frac{1}{3} = \frac{7}{3}$

$f(x) = x^{7/3} - 4\cot(x)\csc(x)$

*ANTIDERIVATIVE OF $\csc(x)\cot(x)$ IS $-\csc(x) + C$

*ANTIDERIVATIVE OF x^n IS $\frac{x^{n+1}}{n+1} + C$

$\Rightarrow \frac{x^{7/3+1}}{7/3+1} - 4(-\csc(x)) + C$ $\quad 7/3 + 1 = 10/3$

$= \frac{x^{10/3}}{10/3} + 4\csc(x) + C = \boxed{\frac{3x^{10/3}}{10} + 4\csc(x) + C}$

XRONOS HW22#6:

FIND THE ANTIDERIVATIVE: $f(t) = \frac{t^2 - 1}{\sqrt{t}}$

$\sqrt{t} = t^{1/2}$

$f(t) = \frac{t^2}{t^{1/2}} - \frac{1}{t^{1/2}}$

$$f(t) = t^{2-1/2} - t^{-1/2}$$

$$f(t) = t^{3/2} - t^{-1/2}$$

* ANTIDERIVATIVE OF x^n IS $\frac{x^{n+1}}{n+1} + c$

$$\Rightarrow \frac{t^{3/2+1}}{3/2+1} - \frac{t^{-1/2+1}}{-1/2+1} + c = \frac{t^{5/2}}{5/2} - \frac{t^{1/2}}{1/2} + c$$

$$= \frac{2t^{5/2}}{5} - 2t^{1/2} + C$$