XRONOS HW 24 #6: IF $-2 \le f(x) \le 5$ ON $[-1,3]$ THEN FIND THE UPPER AND LOWER BOUNDS FOR $\int_{-1}^{3} f(x)\,dx$.

*IF $m \le f(x) \le M$ ON $[a,b]$, THEN $m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$

$-2 \le f(x) \le 5$ ON $[-1,3]$, SO $-2(3-(-1)) \le \int_a^b f(x)\,dx \le 5(3-(-1))$

$$\Rightarrow -2(4) \le \int_a^b f(x)\,dx \le 5(4)$$

$$\Rightarrow -8 \le \int_a^b f(x)\,dx \le 20$$

XRONOS HW 25 #8:

WHAT IS THE AREA UNDER THE CURVE $f(x) = \cos(x) + 2$ ON $[\pi/2, \pi]$?

$$\int_{\pi/2}^{\pi} \cos(x) + 2\,dx = \sin(x) + 2x \Big|_{\pi/2}^{\pi} = [\sin(\pi) + 2(\pi)] - [\sin(\pi/2) + 2(\pi/2)]$$

$$= [0 + 2\pi] - [1 + \pi]$$

$$= 2\pi - 1 - \pi = \boxed{\pi - 1}$$

XRONOS HW 25 #10:

IF $h(x) = \int_0^x t \sin(t)\,dt$, THEN $h'(x) = ?$

BY THE FUNDAMENTAL THEOREM OF CALCULUS, $h'(x) = x\sin(x)$.

XRONOS HW 25 #12:

IF $h(x) = \int_{2x}^{x^2} \cos(t)\sin(t)\,dt$, THEN $h'(x) = ?$

$\downarrow$

$$h(x) = \int_{2x}^{0} \cos(t)\sin(t)\,dt + \int_{0}^{x^2} \cos(t)\sin(t)\,dt$$

$$= -\int_{0}^{2x} \cos(t)\sin(t)\,dt + \int_{0}^{x^2} \cos(t)\sin(t)\,dt$$

$$\Rightarrow \frac{d}{dx}[h(x)] = \frac{d}{dx}\left[\int_{0}^{2x} -\cos(t)\sin(t)\,dt\right] + \frac{d}{dx}\left[\int_{0}^{x^2} \cos(t)\sin(t)\,dt\right]$$

$f(x) = 2x$
$g(x) = x^2$

$$\Rightarrow h'(x) = \frac{d}{dx}\left[\int_{0}^{f(x)} -\cos(t)\sin(t)\,dt\right] + \frac{d}{dx}\left[\int_{0}^{g(x)} \cos(t)\sin(t)\,dt\right]$$

$$\Rightarrow h'(x) = [-\cos(f(x))\sin(f(x))]\left[\frac{df}{dx}\right] + [\cos(g(x))\sin(g(x))]\left[\frac{dg}{dx}\right]$$

$$\Rightarrow h'(x) = -\cos(2x)\sin(2x)(2) + \cos(x^2)\sin(x^2)2x$$

$$= \boxed{-2\cos(2x)\sin(2x) + 2x\cos(x^2)\sin(x^2)}$$

XRONOS HW 25 #13:

IF $h(x) = \int_{\cos(x)}^{\pi} (t+1)^2\,dt$, THEN $h'(x) = ?$

$$h(x) = -\int_{\pi}^{\cos(x)} (t+1)^2\,dt$$

$$= \int_{\pi}^{\cos(x)} -(t+1)^2\,dt$$

IF $F(x) = \int_{\pi}^{x} -(t+1)^2\,dt$, THEN $h(x) = F(\cos(x))$.

NOW, USING THE CHAIN RULE, $h'(x) = F'(\cos(x)) \cdot \frac{d}{dx}(\cos(x))$

$$\Rightarrow h'(x) = -\sin(x)F'(\cos(x))$$

NOTE: $F'(x) = -(t+1)^2 \Rightarrow F'(\cos(x)) = -(\cos(x)+1)^2$

$\Rightarrow h'(x) = -\sin(x) F'(\cos(x)) = -\sin(x)(-\cos(x)+1)^2 = \sin(x)(\cos(x)+1)^2$

XRONOS HW 25 #11:

IF $h(x) = \int_x^2 t + e^t \, dt$, THEN $h'(x) = ?$

$$h(x) = \int_x^2 t + e^t \, dt = -\int_2^x t + e^t \, dt = \int_2^x -t - e^t \, dt$$

$\Rightarrow$ BY THE FUNDAMENTAL THEOREM OF CALCULUS, $h'(x) = -x - e^x$