

MAC2311 Class Number 15498

QUIZ 10

3/28/2019

Name: SOLUTIONS

1. Consider $f(x) = x^2 - 2x + 3$. On what interval is f increasing? On what interval is f decreasing?

(1) FIND CRITICAL POINTS: $f'(x) = 2x - 2$, $f'(x) = 0 \Rightarrow 2x - 2 = 0$
 $2x = 2 \Rightarrow x = 1$

(2) ANALYZE f' AROUND CRITICAL POINTS:

• $x > 1$: $f'(x) = 2x - 2 = 2(x-1)$, WHEN $x > 1$, $x-1 > 0$ $\rightarrow f'(x) > 0 \rightarrow f$ IS INCREASING

• $x < 1$: $f'(x) = 2x - 2 = 2(x-1)$, WHEN $x < 1$, $x-1 < 0$ $\rightarrow f'(x) < 0 \rightarrow f$ IS DECREASING

\therefore INCREASING $(1, \infty)$
 DECREASING $(-\infty, 1)$

2. Find the limit using L'Hopital's Rule.

NOTE: THE LIMIT AS $x \rightarrow \infty$ OF THE NUMERATOR AND DENOMINATOR ARE BOTH ∞ , SO THIS IS THE ∞/∞ INDETERMINANT CASE.

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{x^3 + 15} = \lim_{x \rightarrow \infty} \frac{\frac{d}{dx} [\ln(x)]}{\frac{d}{dx} [x^3 + 15]}$$

L'HOPITAL'S RULE

• $\frac{d}{dx} [\ln(x)] = \frac{1}{x}$ \leftarrow DERIVATIVE OF NUMERATOR

• $\frac{d}{dx} [x^3 + 15] = 3x^2$ \leftarrow DERIVATIVE OF DENOMINATOR

$$\begin{aligned} \rightarrow \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{3x^2} &= \lim_{x \rightarrow \infty} \frac{1}{x} \cdot \frac{1}{3x^2} \\ &= \lim_{x \rightarrow \infty} \frac{1}{3x^3} = \boxed{0} \end{aligned}$$

3. Find the limit using L'Hopital's Rule.

NOTE: THE LIMIT AS $x \rightarrow 1$ OF THE NUMERATOR AND DENOMINATOR ARE BOTH 0, SO THIS IS THE $0/0$ INDETERMINANT CASE

$$\lim_{x \rightarrow 1} \frac{5x^4 - 4x^2 - 1}{10 - t^2 - 9t} = \lim_{x \rightarrow 1} \frac{\frac{d}{dx} [5x^4 - 4x^2 - 1]}{\frac{d}{dx} [10 - t^2 - 9t]}$$

$\frac{d}{dx} [5x^4 - 4x^2 - 1] = 20x^3 - 8x$

$\frac{d}{dx} [10 - t^2 - 9t] = -2t - 9$

$$\rightarrow \lim_{x \rightarrow 1} \frac{20x^3 - 8x}{-2t - 9} = \frac{20(1) - 8(1)}{-2(1) - 9}$$

$$= \frac{12}{-2-9} = \boxed{\frac{-12}{11}}$$