MAC2311 Class Number 15534
QUIZ 10
3/28/2019

Name: SOLUTIONS

1. Consider $f(x) = x^2 - 2x + 37$. On what interval is f increasing? On what interval is f decreasing?

(1) FIND CRITICAL POINTS: $f'(x) = 2x - 2$, $f'(x) = 0 \Rightarrow 2x - 2 = 0 \Rightarrow 2x = 2 \Rightarrow x = 1$

(2) ANALYZE f' AROUND CRITICAL POINTS:

- $x > 1$: $f'(x) = 2x - 2 = 2\underline{(x-1)}$ — WHEN $x > 1$, $x - 1 > 0 \rightarrow f'(x) > 0 \rightarrow f$ IS INCREASING
- $x < 1$: $f'(x) = 2x - 2 = 2\underline{(x-1)}$ — WHEN $x < 1$, $x - 1 < 0 \rightarrow f'(x) < 0 \rightarrow f$ IS DECREASING

$\therefore$ $\boxed{\text{INCREASING } (1,\infty) \quad \text{DECREASING } (-\infty,1)}$

2. Find the limit using L'Hopital's Rule.

NOTE: THE LIMIT AS $x \to \infty$ OF THE NUMERATOR AND DENOMINATOR ARE BOTH ∞, SO THIS IS THE ∞/∞ INDETERMINANT CASE.

$$\lim_{x\to\infty} \frac{\ln(x)}{x^4+5} = \lim_{x\to\infty} \frac{\frac{d}{dx}[\ln(x)]}{\frac{d}{dx}[x^4+5]}$$

L'HOPITAL'S RULE

- $\frac{d}{dx}[\ln(x)] = \frac{1}{x}$ ← DERIVATIVE OF NUMERATOR
- $\frac{d}{dx}[x^4+5] = 4x^3$ ← DERIVATIVE OF DENOMINATOR

$$\rightarrow \lim_{x\to\infty} \frac{\frac{1}{x}}{4x^3} = \lim_{x\to\infty} \frac{1}{x} \cdot \frac{1}{4x^3} = \lim_{x\to\infty} \frac{1}{4x^4} = \boxed{0}$$

3. Find the limit using L'Hopital's Rule.

NOTE: THE LIMIT AS $x \to 1$ OF THE NUMERATOR AND DENOMINATOR ARE BOTH 0, SO THIS IS THE $0/0$ INDETERMINANT CASE

$$\lim_{x\to 1} \frac{5x^3 - 4x^2 - 1}{10 - t^2 - 9t} = \lim_{x\to 1} \frac{\frac{d}{dx}[5x^3 - 4x^2 - 1]}{\frac{d}{dx}[10 - t^2 - 9t]}$$

$\frac{d}{dx}[5x^3 - 4x^2 - 1] = 15x^2 - 8x$

$\frac{d}{dx}[10 - t^2 - 9t] = -2t - 9$

$$\rightarrow \lim_{x\to 1} \frac{15x^2 - 8x}{-2t - 9} = \frac{15(1) - 8(1)}{-2(1) - 9} = \frac{15 - 8}{-2 - 9} = \boxed{-\frac{7}{11}}$$