MAC2311 Class Number 15498
QUIZ 12
4/11/2019

Name: SOLUTIONS

1. Find two positive numbers whose product is 49 and whose sum is the smallest possible.

*WANT: TWO #'S X, Y SO THAT $xy = 49$ AND $\underbrace{x + y = S}_{\text{"SUM"}}$ IS AS SMALL AS POSSIBLE

* X AND Y ARE POSITIVE, SO $x, y > 0$ (IN FACT, $x, y \neq 0$) *ONLY USE +7 B/C $x, y > 0$

*$xy = 49 \Rightarrow y = \frac{49}{x}$ WRITE S AS A FUNCTION OF X: $S(x) = x + y = x + \frac{49}{x} = x + 49x^{-1}$

DERIVE $S(x) = x + 49x^{-1} \Rightarrow S'(x) = 1 - 49x^{-2}$

$S'(x) = 0 \Rightarrow 1 - \frac{49}{x^2} = 0 \Rightarrow 1 = \frac{49}{x^2} \Rightarrow x^2 = 49 \Rightarrow x = \pm 7 \Rightarrow xy = 49 \Rightarrow 7y = 49 \Rightarrow \boxed{y = 7}$

$\boxed{x = 7}$

2. Find the antiderivative of

$$f(x) = \frac{x^4 - x^2 + 4x - 3}{x}$$

*REWRITE $f(x)$ AS $f(x) = \frac{x^4}{x} - \frac{x^2}{x} + \frac{4x}{x} - \frac{3}{x}$

*ANTIDERIVATIVE OF $f(x) = x^3 - x + 4 - 3\left(\frac{1}{x}\right)$: $\frac{x^{3+1}}{3+1} - \frac{x^{1+1}}{1+1} + 4x - 3\ln(x) + C$

$= \boxed{\frac{x^4}{4} - \frac{x^2}{2} + 4x - 3\ln(x) + C}$

3. Determine $f(x)$, given

$$f'(x) = 9x^2 - 4x + 3, \quad f(1) = 8$$

$f(x) =$ ANTIDERIVATIVE OF $f'(x)$

$\Rightarrow f(x) = \frac{9x^{2+1}}{2+1} - \frac{4x^{1+1}}{1+1} + 3x + C$

$f(x) = \frac{9x^3}{3} - \frac{4x^2}{2} + 3x + C$

$f(x) = 3x^3 - 2x^2 + 3x + C$

$f(1) = 8$

$\Rightarrow \underbrace{3(1^3) - 2(1^2) + 3(1) + C}_{f(x)} = 8$

$\Rightarrow 3 - 2 + 3 + C = 8$

$\Rightarrow 4 + C = 8$

$\Rightarrow C = 4$

SO, $\boxed{f(x) = 3x^3 - 2x^2 + 3x + 4}$