

MAC2311 Class Number 15534

QUIZ 12

4/11/2019

Name: SOLUTIONS

1. Find two positive numbers whose product is 49 and whose sum is the smallest possible.

*WANT: TWO #'S x, y SUCH THAT $xy=49$ AND $x+y=S$ IS AS SMALL AS POSSIBLE
 * x AND y ARE POSITIVE, SO $x, y > 0$ (IN FACT, $x, y \neq 0$)
 * $xy=49 \Rightarrow y = \frac{49}{x}$ WRITE S AS A FUNCTION OF x : $S(x) = x+y = x + \frac{49}{x} = x + 49x^{-1}$
 DERIVE $S(x) = x + 49x^{-1} \Rightarrow S'(x) = 1 - 49x^{-2}$

$S'(x)=0 \Rightarrow 1 - \frac{49}{x^2} = 0 \Rightarrow 1 = \frac{49}{x^2} \Rightarrow x^2 = 49 \Rightarrow x = \pm 7 \Rightarrow xy = 49 \Rightarrow 7y = 49 \Rightarrow y = 7$

2. Find the antiderivative of

$$f(x) = \frac{x^4 - x^2 + 3x - 7}{x}$$

*REWRITE $f(x)$ AS $f(x) = \frac{x^4}{x} - \frac{x^2}{x} + \frac{3x}{x} - \frac{7}{x}$

*ANTIDERIVATIVE OF $f(x) = x^3 - x + 3 - 7(\frac{1}{x})$: $\frac{x^{3+1}}{3+1} - \frac{x^{1+1}}{1+1} + 3x - 7 \ln(x) + C$
 $= \frac{x^4}{4} - \frac{x^2}{2} + 3x - 7 \ln(x) + C$

3. Determine $f(x)$, given

$$f'(x) = 9x^2 - 4x + 2, \quad f(1) = 9$$

$f(x) =$ ANTIDERIVATIVE OF $f'(x)$

$$\Rightarrow f(x) = \frac{9x^{2+1}}{2+1} - \frac{4x^{1+1}}{1+1} + 2x + C$$

$$f(x) = \frac{9x^3}{3} - \frac{4x^2}{2} + 2x + C$$

$$f(x) = 3x^3 - 2x^2 + 2x + C$$

$$f(1) = 9$$

$$\Rightarrow 3(1^3) - 2(1^2) + 2(1) + C = 9$$

$$\Rightarrow 3 - 2 + 2 + C = 9$$

$$\Rightarrow 3 + C = 9$$

$$\Rightarrow C = 6$$

$$\text{So, } f(x) = 3x^3 - 2x^2 + 2x + 6$$