

MAC2311 Class Number 15498

QUIZ 8

3/14/2019

Name: SOLUTIONS

1. Compute the differential of the function:

NOTE: $-\frac{4}{(x+12)^2} = -4(x+12)^{-2}$

$$y = -\frac{4}{(x+12)^2}$$

$$dy = f'(x)dx \Rightarrow dy = \underbrace{-4(-2)(x+12)^{-3} \left[\frac{d}{dx}(x+12) \right]}_{f'(x) \text{ USING POWER RULE AND CHAIN RULE}} dx \Rightarrow \boxed{dy = \left(\frac{8}{(x+12)^3} \right) dx}$$

2. Use logarithmic differentiation to find

1. LET $f(x) = \frac{(x^2+1)}{(x+1)^2}$

$$\frac{d}{dx} \left(\frac{(x^2+1)}{(x+1)^2} \right)$$

2. TAKE \ln OF BOTH SIDES:

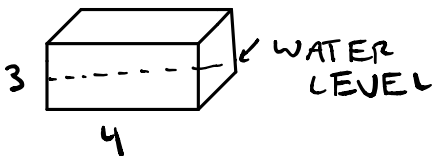
$$\ln(f(x)) = \ln\left(\frac{(x^2+1)}{(x+1)^2}\right) \Rightarrow \ln(f(x)) = \ln(x^2+1) - 2\ln(x+1)$$

PROPERTY OF LOGARITHM

3. TAKE DERIVATIVE OF BOTH SIDES: $\frac{1}{f(x)} f'(x) = \left(\frac{1}{x^2+1} \right) \left(\frac{d}{dx} x^2+1 \right) - 2 \left(\frac{1}{x+1} \right) \left(\frac{d}{dx} x+1 \right)$

4. SOLVE FOR $f'(x)$: $f'(x) = f(x) \left[\frac{2x}{x^2+1} - \frac{2}{x+1} \right] \Rightarrow \boxed{f'(x) = \left(\frac{(x^2+1)}{(x+1)^2} \right) \left(\frac{2x}{x^2+1} - \frac{2}{x+1} \right)}$

3. Water pours into a fish tank at a rate of 2 cubic meters per minute. How fast is the water level rising if the base of the fish tank is a 3 meter by 4 meter rectangle?



1. WANT TO FIND $\frac{dh}{dt}$, h = HEIGHT OF WATER LEVEL

2. GIVEN: $\frac{dV}{dt} = 2 \text{ m}^3/\text{min}$, $l = 4$, $w = 3$
 "LENGTH" "WIDTH"

3. $V = lwh$ VOLUME FORMULA; $V = 4 \cdot 3 \cdot h = 12h$

4. DERIVATIVE: $\frac{dV}{dt} = 12 \frac{dh}{dt}$

5. PLUG IN $\frac{dV}{dt} = 2 \Rightarrow 2 = 12 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{2}{12} \Rightarrow \boxed{\frac{dh}{dt} = \frac{1}{6} \text{ m/min}}$