MAC2311 Class Number 15534
QUIZ 8
3/14/2019

Name: SOLUTIONS

1. Compute the differential of the function:

$$y = -\frac{3}{(x+13)^2}$$

NOTE: $\frac{-3}{(x+13)^2} = -3(x+13)^{-2}$

$dy = f'(x)dx \Rightarrow dy = -3(-2)(x+13)^{-3}\left[\frac{d}{dx}(x+13)\right]dx \Rightarrow \boxed{dy = \left(\frac{6}{(x+13)^3}\right)dx}$

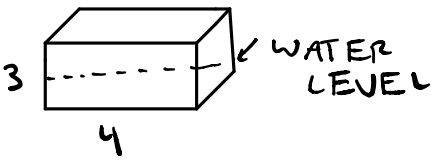
$f'(x)$ USING POWER RULE AND CHAIN RULE

2. Use logarithmic differentiation to find

$$\frac{d}{dx}\left(\frac{(x+1)^2}{(x^3+1)}\right)$$

1. LET $f(x) = \frac{(x+1)^2}{(x^3+1)}$
2. TAKE ln OF BOTH SIDES:
$\ln(f(x)) = \ln\left(\frac{(x+1)^2}{(x^3+1)}\right) \Rightarrow \ln(f(x)) = 2\ln(x+1) - \ln(x^3+1)$ PROPERTY OF LOGARITHM
3. TAKE DERIVATIVE OF BOTH SIDES: $\frac{1}{f(x)}f'(x) = 2\left(\frac{1}{x+1}\right)\left(\frac{d}{dx}(x+1)\right) - \frac{1}{x^3+1}\left(\frac{d}{dx}(x^3+1)\right)$
4. SOLVE FOR $f'(x)$: $f'(x) = f(x)\left[\frac{2}{x+1} - \frac{3x^2}{x^3+1}\right] \Rightarrow \boxed{f'(x) = \left(\frac{(x+1)^2}{(x^3+1)}\right)\left(\frac{2}{x+1} - \frac{3x^2}{x^3+1}\right)}$

3. Water pours into a fish tank at a rate of 2 cubic meters per minute. How fast is the water level rising if the base of the fish tank is a 3 meter by 4 meter rectangle?



1. WANT TO FIND $\frac{dh}{dt}$, h = HEIGHT OF WATER LEVEL
2. GIVEN: $\frac{dV}{dt} = 2\ m^3/min$, $l = 4$, $w = 3$ ("LENGTH", "WIDTH")
3. $V = lwh$ VOLUME FORMULA; $V = 4 \cdot 3 \cdot h = 12h$
4. DERIVATIVE: $\frac{dV}{dt} = 12\frac{dh}{dt}$
5. PLUG IN $\frac{dV}{dt} = 2 \Rightarrow 2 = 12\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{2}{12} \Rightarrow \boxed{\frac{dh}{dt} = \frac{1}{6}\ m/min}$