LIMITS AT INFINITY:

*NOTE: $\lim_{x\to\pm\infty} \frac{1}{x} = 0$ *

METHOD ONE: DIVIDE TOP AND BOTTOM BY HIGHEST POWER OF THE NUMERATOR

EXAMPLE:

1. $\lim_{x\to\infty} \frac{2x^2+13}{x^2-3x+2} = \lim_{x\to\infty} \frac{2x^2/x^2 + 13/x^2}{x^2/x^2 - 3x/x^2 + 2/x^2} = \lim_{x\to\infty} \frac{2 + 13/x^2}{1 - 3/x + 2/x^2}$

$= \frac{2}{1} = \boxed{2}$

METHOD TWO: USE THE FOLLOWING RULES:

1. IF THE HIGHEST POWER OF THE NUMERATOR IS EQUAL TO THE HIGHEST POWER OF THE DENOMINATOR, THEN THE LIMIT IS THE RATIO OF THE LEADING COEFFICIENTS
2. IF THE HIGHEST POWER OF THE NUMERATOR IS GREATER THAN THE HIGHEST POWER OF THE DENOMINATOR, THEN THE LIMIT IS $\pm\infty$ (NO HORIZONTAL ASYMPTOTE)
3. IF THE HIGHEST POWER OF THE NUMERATOR IS LESS THAN THE HIGHEST POWER OF THE DENOMINATOR, THEN THE LIMIT IS 0

FINDING HORIZONTAL ASYMPTOTES:

TO FIND HORIZONTAL ASYMPTOTES OF A RATIONAL FUNCTION INVOLVING RADICALS, EVALUATE THE LIMIT AT $+\infty$ AND $-\infty$.

NOTE: $\sqrt{x^2} = |x|$

EXAMPLE: FIND HORIZONTAL ASYMPTOTES OF THE FUNCTION

$f(x) = \frac{3x}{\sqrt{4x^2+2}}$:

AS $x \to +\infty$, $x > 0$ SO $|x| = x$

1. $\lim_{x \to +\infty} \frac{3x}{\sqrt{4x^2+2}} = \lim_{x \to +\infty} \frac{3x / |x|}{\sqrt{4x^2+2} / \sqrt{x^2}} = \lim_{x \to +\infty} \frac{3x/x}{\sqrt{\frac{4x^2+2}{x^2}}}$

$= \lim_{x \to +\infty} \frac{3}{\sqrt{4 + 2/x^2}} = \frac{3}{\sqrt{4}} = \frac{3}{2}$

0

AS $x \to -\infty$, $x < 0$, SO $|x| = -x$

2. $\lim_{x \to -\infty} \frac{3x}{\sqrt{4x^2+2}} = \lim_{x \to -\infty} \frac{3x / |x|}{\sqrt{4x^2+2} / \sqrt{x^2}} = \lim_{x \to -\infty} \frac{3x / -(x)}{\sqrt{\frac{4x^2+2}{x^2}}}$

$= \lim_{x \to -\infty} \frac{-3}{\sqrt{4 + 2/x^2}} = \frac{-3}{\sqrt{4}} = \frac{-3}{2}$

0

SO, HORIZONTAL ASYMPTOTES ARE $y = 3/2$, $y = -3/2$

EXPONENTIAL AND LOG FUNCTIONS:

WHEN $a > 1$:

1. $\lim_{x \to \infty} a^x = \infty$
2. $\lim_{x \to -\infty} a^x = 0$
3. $\lim_{x \to \infty} a^{-x} = 0$
4. $\lim_{x \to -\infty} a^{-x} = \infty$

WHEN $0 < a < 1$:

1. $\lim_{x \to \infty} a^x = 0$
2. $\lim_{x \to -\infty} a^x = \infty$
3. $\lim_{x \to \infty} a^{-x} = \infty$
4. $\lim_{x \to -\infty} a^{-x} = 0$

LOG & ln:

1. $\lim_{x \to \infty} \ln x = \infty$
2. $\lim_{x \to 0^+} \ln x = -\infty$
3. $\lim_{x \to \infty} \log_a x = \infty$
4. $\lim_{x \to 0^+} \log_a x = -\infty$

AVERAGE AND INSTANTANEOUS VELOCITY:

AVERAGE VELOCITY ON $[a,b]$: $V_{AVE} = \frac{s(b) - s(a)}{b - a}$

INSTANTANEOUS VELOCITY AT X: $V = \lim_{t \to x} \frac{S(t) - S(x)}{t - x}$

THE DERIVATIVE (SLOPE OF A TANGENT LINE):

LIMIT DEFINITION OF DERIVATIVE:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$\lim_{t \to x} \frac{f(t) - f(x)}{t - x} = f'(x)$$

*USE DERIVATIVE TO FIND SLOPE OF TANGENT LINES, EQUATIONS OF TANGENT LINES, INSTANTANEOUS RATE OF CHANGE AT A SPECIFIC POINT

REMARK: (1) IF A FUNCTION IS DIFFERENTIABLE, THEN IT IS CONTINUOUS

(2) A HORIZONTAL TANGENT LINE $\Rightarrow f'(x) = 0$

(3) A VERTICAL TANGENT LINE $\Rightarrow f'(x)$ IS UNDEFINED

(4) f IS DIFFERENTIABLE AT A POINT, x, IF f IS CONTINUOUS AT x, f HAS NO "SHARP TURNS" AT x, AND IF f DOES NOT HAVE A VERTICAL TANGENT LINE AT x