CHAIN RULE:

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\,g'(x)$$

EXAMPLES:

(1) $\frac{d}{dx}\left(f(x)\right)^n = n f(x)^{n-1} \cdot f'(x)$

(2) $\frac{d}{dx} e^{f(x)} = f'(x)\, e^{f(x)}$

(3) $\frac{d}{dx}\left(\ln(f(x))\right) = \frac{1}{f(x)} \cdot f'(x)$

(4) $\frac{d}{dx}\left[\sin(f(x))\right] = f'(x)\cos(f(x))$

INVERSE TRIG DERIVATIVES:

(1) $\frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\sqrt{1-x^2}}$

(2) $\frac{d}{dx}\left[\arccos(x)\right] = \frac{-1}{\sqrt{1-x^2}}$

(3) $\frac{d}{dx}\left[\arctan(x)\right] = \frac{1}{1+x^2}$

(4) $\frac{d}{dx}\left[\operatorname{arccsc}(x)\right] = \frac{-1}{|x|\sqrt{x^2-1}}$

(5) $\frac{d}{dx}\left[\operatorname{arcsec}(x)\right] = \frac{1}{|x|\sqrt{x^2-1}}$

(6) $\frac{d}{dx}\left[\operatorname{arccot}(x)\right] = \frac{-1}{1+x^2}$

IMPLICIT DIFFERENTIATION:

EXAMPLES:

1. IMPLICITLY DERIVE $y^2 + 2x = x^2$ AND SOLVE FOR $\frac{dy}{dx}$:

$$2y\left[\frac{dy}{dx}\right] + 2 = 2x$$

$$\Rightarrow 2y\left[\frac{dy}{dx}\right] = 2x - 2 \Rightarrow \frac{dy}{dx} = \frac{2(x-1)}{2y} = \frac{x-1}{y}$$

2. IMPLICITLY DERIVE $y - y^3 = 7x^3$ AND SOLVE FOR $\frac{dy}{dx}$:

$$\Rightarrow \frac{dy}{dx} - 3y^2\frac{dy}{dx} = 21x^2$$

$$\Rightarrow \frac{dy}{dx}\left[1 - 3y^2\right] = 21x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{21x^2}{1 - 3y^2}$$