

### CRITICAL NUMBERS:

THE VALUES OF  $x$  THAT ARE IN THE DOMAIN OF  $f(x)$  AND OCCUR WHEN  $f'(x) = 0$  AND  $f'(x)$  IS UNDEFINED.

### TO FIND CRITICAL NUMBERS:

(1) FIND  $f'(x)$

(2) SOLVE  $f'(x) = 0$  FOR  $x$

(3) DETERMINE WHEN  $f'(x)$  IS UNDEFINED.

### THE EXTREME VALUE THEOREM:

IF  $f$  IS A CONTINUOUS FUNCTION OVER THE CLOSED, BOUNDED INTERVAL  $[a, b]$ , THEN THERE IS A POINT IN  $[a, b]$  AT WHICH  $f$  HAS AN ABSOLUTE MAXIMUM OVER  $[a, b]$ , AND THERE IS A POINT IN  $[a, b]$  AT WHICH  $f$  HAS AN ABSOLUTE MINIMUM OVER  $[a, b]$ .

### ROLLE'S THEOREM:

IF  $f$  IS CONTINUOUS OVER  $[a, b]$ , DIFFERENTIABLE OVER  $(a, b)$ , AND IF  $f(a) = f(b)$ , THEN THERE EXISTS AT LEAST ONE  $c$  IN  $(a, b)$  SUCH THAT  $f'(c) = 0$ .

### THE MEAN VALUE THEOREM:

IF  $f$  IS CONTINUOUS OVER  $[a, b]$  AND DIFFERENTIABLE OVER  $(a, b)$ , THEN THERE EXISTS AT LEAST ONE POINT  $c$  IN  $(a, b)$  SUCH THAT  $f'(c) = \frac{f(b) - f(a)}{b - a}$ .

TO FIND  $c$  SUCH THAT THE MEAN VALUE THEOREM HOLDS,

(1) FIND  $f(b)$  AND  $f(a)$

(2) PLUG IN TO  $\frac{f(b) - f(a)}{b - a}$  AND SIMPLIFY

(3) SET  $f'(c)$  EQUAL TO YOUR SOLUTION IN (2) AND SOLVE FOR  $c$ .

## LOCAL MAX AND MIN:

IF  $f$  HAS A LOCAL EXTREMUM AT  $c$  AND  $f$  IS DIFFERENTIABLE AT  $c$ , THEN

$$f'(c) = 0.$$

TO FIND LOCAL EXTREMA:

(1) EVALUATE  $f$  AT THE ENDPOINTS OF INTERVAL GIVEN

(2) FIND  $c$  SUCH THAT  $f'(c) = 0$

(3) ANALYZE RESULTS OF (1) AND (2) TO DETERMINE MIN/MAX

NOTE:

\* IF  $f'(c) = 0$  AND  $f''(c) > 0 \Rightarrow f$  HAS A LOCAL MINIMUM AT  $c$

\* IF  $f'(c) = 0$  AND  $f''(c) < 0 \Rightarrow f$  HAS A LOCAL MAXIMUM AT  $c$