INTERVAL ON WHICH A FUNCTION IS INCREASING OR DECREASING:

$f(x)$ IS INCREASING IF $f'(x) > 0$

$f(x)$ IS DECREASING IF $f'(x) < 0$

CONCAVITY AND POINTS OF INFLECTION:

IF f' IS INCREASING OVER AN INTERVAL I, THEN f IS CONCAVE UP OVER I.

IF f' IS DECREASING OVER AN INTERVAL I, THEN f IS CONCAVE DOWN OVER I.

- IF $f''(x) > 0$ FOR ALL $x \in [a,b]$ $\Rightarrow$ f IS CONCAVE UP OVER $[a,b]$
- IF $f''(x) < 0$ FOR ALL $x \in [a,b] \Rightarrow f$ IS CONCAVE DOWN OVER $[a,b]$.

IF f IS CONTINUOUS AT A POINT x AND IF f CHANGES CONCAVITY AT x, THEN $(x, f(x))$ IS AN INFLECTION POINT OF f.

TO FIND INFLECTION POINTS:

(1) DETERMINE WHERE f CHANGES CONCAVITY

(2) EVALUATE f AT THE VALUES FOUND IN 1

L'HOPITAL'S RULE:

SUPPOSE THAT f AND g ARE DIFFERENTIABLE AND $g'(x) \neq 0$ NEAR a. SUPPOSE THAT EITHER

(1) $\lim_{x \to a} f(x) = 0$ AND $\lim_{x \to a} g(x) = 0$

OR

(2) $\lim_{x \to a} f(x) = \pm\infty$ AND $\lim_{x \to a} g(x) = \pm\infty$.

THEN, $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ IF IT EXISTS, OR IS $\pm\infty$.

↑THIS IS INDETERMINATE FORM $0/0$ OR ∞/∞.

L'HOPITAL'S RULE APPLIED TO INDETERMINANT PRODUCTS:

IF $\lim_{x\to a} f(x) = 0$ AND $\lim_{x\to a} g(x) = \pm\infty$, THEN TO FIND $\lim_{x\to a} f(x)g(x)$, WE FIND

$\lim_{x\to a} \frac{g(x)}{1/f(x)}$. (THIS IS OK BECAUSE $\frac{g(x)}{1/f(x)} = g(x) \cdot \frac{f(x)}{1} = g(x)f(x)$)

↑

NOW, THIS IS INDETERMINANT FORM "$\frac{\infty}{\infty}$"

L'HOPITAL'S RULE APPLIED TO INDETERMINANT DIFFERENCES:

IF $\lim_{x\to a} f(x) = \infty$ AND $\lim_{x\to a} g(x) = \infty$, THEN TO FIND $\lim_{x\to a} f(x) - g(x)$, REWRITE $f(x) - g(x)$

AS A SINGLE FRACTION, AND THEN APPLY L'HOPITAL'S RULE.

L'HOPITAL'S RULE APPLIED TO INDETERMINANT POWERS:

WANT TO FIND $\lim_{x\to a} f(x)^{g(x)}$, BUT WE OBTAIN THE INDETERMINANT FORM ∞^0, 0^0, OR 1^0. THEN, FOLLOW THE STEPS BELOW TO FIND THE LIMIT:

(1) LET $y = f(x)^{g(x)}$

(2) TAKE ln OF BOTH SIDES AND USE PROPERTIES OF ln TO SIMPLIFY:

$\ln(y) = \ln(f(x)^{g(x)}) = g(x)\ln(f(x))$

(3) TAKE THE LIMIT AS $x \to a$ OF BOTH SIDES:

$\lim_{x\to a}[\ln(y)] = \underbrace{\lim_{x\to a}[g(x)\ln(f(x))]}$

(4) USE AN APPROPRIATE APPLICATION OF L'HOPITAL'S RULE TO FIND THE LIMIT OF $\lim_{x\to a}[g(x)\ln(f(x))]$. SAY YOU FIND THAT $\lim_{x\to a}[g(x)\ln(f(x))] = L$ (L CAN BE $\pm\infty$).

(5) SINCE $\lim_{x\to a}[g(x)\ln(f(x))] = L$, WE HAVE $\lim_{x\to a}[\ln(y)] = L$.

(6) SINCE ln IS CONTINUOUS, REWRITE AS FOLLOWS:

$$\lim_{x \to a} [\ln(y)] = L \Rightarrow \ln\left[\lim_{x \to a} (y)\right] = L.$$

(7). SOLVE $\ln\left[\lim_{x \to a} (y)\right] = L$ AS FOLLOWS:

$$e^{\ln\left[\lim_{x \to a} (y)\right]} = e^{L}$$

$$\lim_{x \to a} (y) = e^{L}$$

THEN, SINCE $y = f(x)^{g(x)}$, WE HAVE $\lim_{x \to a} f(x)^{g(x)} = e^{L}$.