LEFT-ENDPOINT APPROXIMATION:

n = # OF SUBINTERVALS (GIVEN IN THE PROBLEM)

$\Delta x = \frac{b-a}{n}$

$L_n = f(x_0)\Delta x + f(x_1)\Delta x + \dots + f(x_{n-1})\Delta x$

LEFT ENDPOINT APPROX

RIGHT-ENDPOINT APPROXIMATION:

n = # OF SUBINTERVALS (GIVEN IN THE PROBLEM)

$\Delta x = \frac{b-a}{n}$

$R_n = f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$

RIGHT ENDPOINT APPROX

MIDPOINT APPROXIMATION:

n = # OF SUBINTERVALS (GIVEN IN THE PROBLEM)

x_i^m - MIDPOINT OF SUBINTERVALS

$\Delta x = \frac{b-a}{n}$

$M_n = f(x_1^m)\Delta x + f(x_2^m)\Delta x + \dots + f(x_n^m)\Delta x$

MIDPOINT APPROX.

THE DEFINITE INTEGRAL:

f(x) - FUNCTION DEFINED ON [a,b]

$$\Rightarrow \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,dx$$

PROPERTIES OF THE DEFINITE INTEGRAL:

1. $\int_a^a f(x)dx = 0$

2. $\int_b^a f(x)dx = -\int_a^b f(x)dx$

3. $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

4. IF C IS A CONSTANT, $\int_a^b c\, f(x)dx = c\int_a^b f(x)dx$

5. $\int_a^b f(x)dx = \int_a^c f(x)\,dx + \int_c^b f(x)dx$ FOR $a < c < b$

THE COMPARISON THEOREM:

1. IF $f(x) \geq 0$ FOR $a \leq x \leq b \Rightarrow \int_a^b f(x)dx \geq 0$

2. IF $f(x) \geq g(x)$ FOR $a \leq x \leq b \Rightarrow \int_a^b f(x)dx \geq \int_a^b g(x)dx$

3. IF m AND M ARE CONSTANTS SUCH THAT $m \leq f(x) \leq M$ FOR $a \leq x \leq b$
$\Rightarrow m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$

AREA AND THE INTEGRAL:

A_1 - AREA BETWEEN $f(x)$ AND X-AXIS THAT LIES ABOVE X-AXIS

A_2 - AREA BETWEEN $f(x)$ AND X-AXIS THAT LIES BELOW X-AXIS

f - INTEGRABLE FUNCTION ON $[a,b]$

$\Rightarrow$ NET SIGNED AREA $= \int_a^b f(x)dx = A_1 - A_2$

$\Rightarrow$ TOTAL AREA $= \int_a^b |f(x)|dx = A_1 + A_2$

THE FUNDAMENTAL THEOREM OF CALCULUS (PART 1):

IF $f(x)$ IS CONTINUOUS OVER $[a,b]$ AND THE FUNCTION $F(x)$ IS DEFINED BY $F(x) = \int_a^x f(t)\,dt$, THEN $F'(x) = f(x)$ OVER $[a,b]$.

THE FUNDAMENTAL THEOREM OF CALCULUS (PART 2):

IF $f(x)$ IS CONTINUOUS OVER $[a,b]$ AND $F(x)$ IS ANY ANTIDERIVATIVE OF $f(x)$, THEN $\int_a^b f(x)\,dx = F(b) - F(a)$.

NOTE: NOW YOU CAN CALCULATE AREA UNDER A CURVE USING THE FUNDAMENTAL THEOREM OF CALCULUS (PART 2).

THE NET CHANGE THEOREM:

THE NEW VALUE OF A CHANGING QUANTITY EQUALS THE INITIAL VALUE PLUS THE INTEGRAL OF THE RATE OF CHANGE:

- $F(b) = F(a) + \int_a^b F'(x)\,dx$
- $\int_a^b F'(x)\,dx = F(b) - F(a)$

EVEN / ODD FUNCTIONS:

- IF $f(-x) = f(x)$ THEN f IS SAID TO BE EVEN
- IF $f(-x) = -f(x)$ THEN f IS SAID TO BE ODD

INTEGRATING EVEN FUNCTIONS:

IF $f(x)$ IS A CONTINUOUS EVEN FUNCTION, THEN

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

INTEGRATING ODD FUNCTIONS:

IF $f(x)$ IS A CONTINUOUS ODD FUNCTION, THEN

$$\int_{-a}^{a} f(x)\,dx = 0$$

SUBSTITUTION (DEFINITE INTEGRALS):

LET $u = g(x)$ AND LET g' BE CONTINUOUS OVER $[a,b]$. LET f BE CONTINUOUS OVER THE RATE OF CHANGE OF $u = g(x)$. THEN: $\int_{a}^{b} f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$

INTEGRATE USING U-SUBSTITUTION:

1. DEFINE $u = g(x)$
2. TAKE DERIVATIVE AND SOLVE FOR dx: $du = g'(x)\,dx \Rightarrow \boxed{dx = \dfrac{du}{g'(x)}}$
3. RE-EVALUATE ENDPOINTS $[a,b]$: FOR $x = a$, $u = g(a)$; FOR $x = b$, $u = g(b)$.
4. SUBSTITUTE AND INTEGRATE:

$$\int_{a}^{b} f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,g'\cancel{(x)}\left(\frac{du}{g'\cancel{(x)}}\right) = \int_{g(a)}^{g(b)} f(u)\,du$$