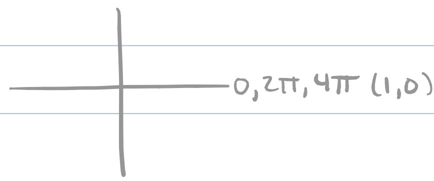
XRONOS HW 3 #4:

$$\lim_{x \to -4} 4(x^3 - 8x^2 + 20x - 16)\tan(\pi x)$$

RECALL THAT $\tan(x)$ IS AN ODD FUNCTION, SO $\tan(-4\pi) = -\tan(4\pi)$.



$$-\tan(4\pi) = -\frac{0}{1} = 0$$

SO, $\lim_{x \to -4} 4(x^3 - 8x^2 + 20x - 16)\tan(\pi x) = 4((-4)^3 - 8(-4)^2 + 20(-4) - 16)(0)$

$$= \boxed{0}$$

XRONOS HW 3 #6:

$$\lim_{x \to 4} \frac{3(\ln(9x+10)+1)}{4((2x+2)^{1/3}+1)} = \frac{3(\ln(9(4)+10)+1)}{4((2(4)+2)^{1/3}+1)}$$

$$= \frac{3(\ln(46)+1)}{4((10)^{1/3}+1)}$$

XRONOS HW 3 #13:

$$\lim_{x \to -5} \frac{-8|x|+40}{2x+10}$$

WHETHER $x \to -5^+$ OR $x \to -5^-$, $x < 0$ SINCE $-5 < 0$. SO, IN EITHER CASE, $|x| = -x$.

$$\Rightarrow \lim_{x \to -5} \frac{-8|x|+40}{2x+10} = \lim_{x \to -5} \frac{-8(-x)+40}{2(x+5)}$$

$$= \lim_{x \to -5} \frac{8x+40}{2(x+5)} = \lim_{x \to -5} \frac{8\cancel{(x+5)}}{2\cancel{(x+5)}} = \frac{8}{2} = \boxed{4}$$

XRONOS HW 3 #18: DOES THE INTERMEDIATE VALUE THEOREM HOLD

FOR $f(x) = \begin{cases} 4e^{x-5} - 3 & -\infty < x < 5 \\ 3e^{x-5} - 3 & 5 \le x < \infty \end{cases}$ ON THE INTERVAL $[-3, 11]$?

1. IS f CONTINUOUS ON [-3,11]? CHECK $\lim_{x\to 5^+} f(x)$ AND $\lim_{x\to 5^-} f(x)$

(SINCE 5 IS OUR "CHANGE POINT"

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} 3e^{x-5} - 3 = 3e^{5-5} - 3 = 3 - 3 = 0$$

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} 4e^{x-5} - 3 = 4e^{5-5} - 3 = 4 - 3 = 0$$

WE HAVE A JUMP DISCONTINUITY AT X=5 AND SINCE 5 IS A POINT IN THE INTERVAL [-3,11], THE INTERMEDIATE VALUE THEOREM DOES NOT HOLD.

XRONOS HW 3 #5:

$$\lim_{x\to 1} \frac{4(x^2-1)}{x-1} = \lim_{x\to 1} \frac{4\cancel{(x-1)}(x+1)}{\cancel{x-1}} = \lim_{x\to 1} 4(x+1) = 4(1+1) = 4(2) = \boxed{8}$$

XRONOS HW 3 #3:

IS $f(x) = \begin{cases} \sqrt{x^2-4} - 9 & -\infty < x \le 2 \\ \dfrac{x^2-13x+22}{x-2} & 2 < x < \infty \end{cases}$ CONTINUOUS AT $x=2$?

CHECK LIMIT OF f AT THE "CHANGE POINT", $x=2$:

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \frac{x^2-13x+22}{x-2} = \lim_{x\to 2^+} \frac{(x-11)\cancel{(x-2)}}{\cancel{x-2}} = 2 - 11 = -9$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \sqrt{x^2-4} - 9 = \sqrt{(2)^2-4} - 9 = 0 - 9 = -9$$

SO, $\lim_{x\to 2^+} f(x) = -9 = \lim_{x\to 2^-} f(x) \Rightarrow$ f IS CONTINUOUS AT $x=2$.

XRONOS HW 3 #1:

$$\lim_{x\to -1} \frac{-5x^3-5}{x^2-5x-6} = \lim_{x\to -1} \frac{-5(x^3+1)}{(x-6)(x+1)} = \lim_{x\to -1} \frac{-5\cancel{(x+1)}(x^2-x+1)}{(x-6)\cancel{(x+1)}}$$

RECALL: (a^3+b^3)

$= (a+b)(a^2-ab+b^2)$

$$= \lim_{x\to -1} \frac{-5(x^2-x+1)}{(x-6)}$$

$$= \frac{-5((-1)^2-(-1)+1)}{-1-6}$$

$$= \frac{-15}{-7} = \boxed{\frac{15}{7}}$$

XRONOS HW 3 #12:

$$\lim_{x\to -8} \frac{-5x-40}{|x+8|} = \lim_{x\to -8} \frac{-5(x+8)}{|x+8|}$$

1. $\lim_{x\to -8^+} \frac{-5(x+8)}{|x+8|}$ SINCE $x \to -8^+$, $x > -8 \Rightarrow x+8>0$

$$\Rightarrow \lim_{x\to -8^+} \frac{-5(x+8)}{(x+8)} = -5$$

2. $\lim_{x\to -8^-} \frac{-5(x+8)}{|x+8|}$ SINCE $x \to -8^-$, $x < -8 \Rightarrow x+8<0$

$$\Rightarrow \lim_{x\to -8^-} \frac{-5\cancel{(x+8)}}{-\cancel{(x+8)}} = \frac{-5}{-1} = 5$$

SO, SINCE $\lim_{x\to -8^+} \frac{-5x-40}{|x+8|} \neq \lim_{x\to -8^-} \frac{-5x-40}{|x+8|}$, $\lim_{x\to 8} \frac{-5x-40}{|x+8|} = \boxed{\text{DNE}}$

XRONOS HW 3 #15:

$$\lim_{x\to 8} (-5x+14)\sqrt{68-x^2} = (-5(8)+14)\sqrt{68-(8)^2}$$

$$= (-40+14)\sqrt{68-64}$$

$$= (-31)\sqrt{4}$$

$$= (-31)(2) = \boxed{-62}$$

XRONOS HW 3 #16:

$$\lim_{x\to 1} e^{(-9x^5-4x^4-3x^2)} = e^{(-9(1)^5-4(1)^4-3(1)^2)} = e^{(-9-4-3)} = \boxed{e^{-16}}$$

*THE DOMAIN OF $f(x)=e^x$ IS ALL REAL NUMBERS

XRONOS HW 3 #17:

DOES THE INTERMEDIATE VALUE THEOREM HOLD FOR

$$f(x)=\begin{cases} 10(x-4)^3-2 & -\infty < x < 5 \\ (x-4)^{1/7}+7 & 5 \le x < \infty \end{cases}$$ ON THE INTERVAL $[0,7]$?

1. IS f CONTINUOUS? CHECK "CHANGE" POINT, $x=5$:

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} (x-4)^{1/7}+7 = (5-4)^{1/7}+7 = 1+7=8$$

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} 10(x-4)^3-2 = 10(5-4)^3-2 = 10-2=8$$

$\Rightarrow$ f IS CONTINUOUS!

2. DOES f CHANGE SIGNS FOR $f(0)$ AND $f(7)$?

$f(0)=10(0-4)^3-2 = 10(-4)^3-2 = -640-2 = -642 < 0$

$f(7)=(7-4)^{1/7}+7 = 3^{1/7}+7 > 0$

SINCE f IS CONTINUOUS ON $[0,7]$ AND SINCE $f(0)<0$ AND $f(7)>0$, THE INTERMEDIATE VALUE THEOREM HOLDS.

XRONOS HW 3 #19:

FIND THE NUMBERS AT WHICH $f(x)=\begin{cases} (x-2)^2-22 & -\infty < x \le -3 \\ x+5 & -3 < x \le -1 \\ x^2-2x+1 & -1 < x < \infty \end{cases}$

IS DISCONTINUOU

LOOK AT "CHANGE POINTS":

1. $\lim_{x \to -3^-} f(x) = (-3-2)^2 - 22$, $\lim_{x \to -3^+} f(x) = -3+5 = 2$

$= (-5)^2 - 22$

$= 25 - 22 = 3$

$\boxed{f \text{ IS DISCONTINUOUS AT } x=-3}$

2. $\lim_{x \to -1^-} f(x) = -1+5 = 4$, $\lim_{x \to -1^+} f(x) = (-1)^2 - 2(-1) + 1$

$= 1 + 2 + 1 = 4$

f IS CONTINUOUS FROM THE LEFT AT $x=-3$ BECAUSE f IS DEFINED AT $x=-3$ FROM THE LEFT. f IS NOT CONTINUOUS FROM THE RIGHT AT $x=-3$.

XRONOS HW 3 #20:

FIND THE NUMBERS AT WHICH $f(x) = \begin{cases} (x-1)^2 & x < 1 \\ \sin(x) & 1 \le x \le 3 \\ x^2 - 2x + 1 & x > 3 \end{cases}$ IS DISCONTINUOUS.

*CHECK THE LIMIT OF f AT THE "CHANGE" POINTS, $x=1$, $x=3$

1. $\lim_{x \to 1^+} f(x) = \sin(1)$, $\lim_{x \to 1^-} f(x) = (1-1)^2 = 0$

SINCE $\sin(1) \neq 0$, WE HAVE THAT $\boxed{f \text{ IS DISCONTINUOUS AT } x=1.}$

2. $\lim_{x \to 3^+} f(x) = 3^2 - 2(3) + 1 = 4$, $\lim_{x \to 3^-} f(x) = \sin(3)$

SINCE $\sin(3) \neq 4$, WE HAVE THAT $\boxed{f \text{ IS DISCONTINUOUS AT } x=3.}$

f IS CONTINUOUS FROM THE RIGHT AT $x=1$ AND DISCONTINUOUS FROM THE RIGHT AT $x=3$. f IS CONTINUOUS FROM THE LEFT AT $x=3$ AND DISCONTINUOUS FROM THE LEFT AT $x=1$.

XRONOS HW 3 #22:

DOES $\sin(x-4) = -x+13$ HAVE A SOLUTION IN $(4, 13)$?

THIS FUNCTION IS CONTINUOUS. REWRITE AS $\sin(x-4)+x-13=0$.

$f(4) = \sin(4-4)+4-13 = 0+4-13 = -9 < 0$

$f(13) = \sin(13-4)+13-13 = \sin(9) > 0$

SINCE THIS FUNCTION IS POSITIVE AT $x=13$ AND NEGATIVE AT $x=4$, $\boxed{\text{THIS FUNCTION HAS A SOLUTION}}$ BY THE INTERMEDIATE VALUE THEOREM.