XRONOS HW4 #1:

$$\lim_{x\to\infty} \frac{\sqrt{49x^2-4}+3}{x}$$

METHOD ONE: THE DEGREE OF THE NUMERATOR IS EQUAL TO THE DEGREE OF THE DENOMINATOR, SO THE LIMIT IS THE RATIO OF LEADING COEFFICIENTS, $\frac{\sqrt{49}}{1} = \frac{7}{1} = \boxed{7}$

METHOD TWO: $\lim_{x\to\infty} \frac{\sqrt{49x^2-4}/\sqrt{x^2} + 3/|x|}{x/|x|}$ * BECAUSE $\sqrt{x^2} = |x|$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{49x^2-4}{x^2}} + 0}{1}$$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{49\cancel{x^2}}{\cancel{x^2}} - \frac{4}{\cancel{x^2}}\to 0}}{1} = \frac{\sqrt{49}}{1} = \boxed{7}$$

XRONOS HW4 #3:

$$\lim_{x\to 9^+} \ln((x+1)(x-9)) = \ln((9+1)(9^+-9)) = \ln(0^+) = \boxed{-\infty}$$

*RECALL: $\lim_{x\to 0^+} \ln(x) = -\infty$

XRONOS HW4 #4:

$$\lim_{x\to 7^+} \frac{3}{x-7} - \ln(x-7)$$

$$= \underbrace{\lim_{x\to 7^+} \frac{3}{x-7}} - \underbrace{\lim_{x\to 7^+} \ln(x-7)}$$

$= \frac{\text{"}3\text{"}}{\#}$, SINCE $x \to 7^+$, CHOOSE $x = 7.1$, $\Rightarrow \frac{3}{7.1-7} = \frac{\text{POSITIVE \#}}{\text{POSITIVE \#}}$

$\Rightarrow +\infty$

$\lim_{x \to 7^+} \ln(x-7) = \ln(0^+) = -\infty$

$\Rightarrow \infty - (-\infty) = \boxed{\infty}$

XRONOS HW4 #7:

FIND VERTICAL ASYMPTOTES, HORIZONTAL ASYMPTOTES, AND HOLES OF

$f(x) = \dfrac{x^2 - 7x + 12}{x^3 - 5x^2 + 3x + 9}$

TO SIMPLIFY $f(x)$, FACTOR NUMERATOR AND DENOMINATOR.

TO FACTOR $\underbrace{x^3} - 5x^2 + 3x + \underbrace{9}$, FIND POSSIBLE RATIONAL ZEROS:

POSSIBLE RATIONAL ZEROS: $\dfrac{\pm 9}{\pm 1}, \dfrac{\pm 3}{\pm 1}, \dfrac{\pm 1}{\pm 1}$

$= 9, -9, 3, -3, 1, -1$

TRY $x=-1$: $f(-1) = (-1)^3 - 5(-1) + 3(-1) + 9 = 0$ ✓, -1 IS A ZERO.

$$\Rightarrow \begin{array}{r|rrrr} -1 & 1 & -5 & 3 & 9 \\ & \downarrow & -1 & 6 & -9 \\ \hline & 1 & -6 & 9 & 0 \\ & \uparrow & \uparrow & \uparrow & \uparrow \text{REMAINDER} \\ & x^2 & x & \text{CONSTANT} & \end{array}$$

$\Rightarrow x^3 - 5x^2 + 3x + 9 = (x+1)(x^2 - 6x + 9) = (x+1)(x-3)^2$

$\Rightarrow f(x) = \dfrac{x^2 - 7x + 12}{x^3 - 5x^2 + 3x + 9} = \dfrac{(x-4)(x-3)}{(x+1)(x-3)^2} = \dfrac{(x-4)}{(x+1)(x-3)}$

$\Rightarrow$ NO HOLES

$\Rightarrow$ VERTICAL ASYMPTOTE: $x=-1$, $x=3$

$\Rightarrow$ H.A.: $y=0$

DEGREE OF NUMERATOR IS SMALLER THAN DEGREE OF DENOMINATOR

XRONOS HW4 #5: FIND VERTICAL ASYMPTOTES, HORIZONTAL ASYMPTOTES, AND HOLES OF $f(x)=\dfrac{x^2+2x}{x^2-4}$

$$f(x)=\frac{x^2+2x}{x^2-4}=\frac{x\cancel{(x+2)}}{\cancel{(x+2)}(x-2)}$$

$$f(x)=\frac{x}{x-2}$$

$\boxed{\text{VERTICAL ASYMPTOTE AT } x=2}$ $(x-2=0 \Rightarrow x=2)$

$\Rightarrow$ HOLE @ $x=-2$ $(x+2=0,\ x=-2)$

$$f(x)=\frac{x}{x-2} \Rightarrow f(-2)=\frac{-2}{-2-2}=\frac{-2}{-4}$$

$$f(-2)=\frac{1}{2}$$

$\boxed{\text{HOLE @ } \left(-2, \tfrac{1}{2}\right)}$

TO FIND HORIZONTAL ASYMPTOTE:

$$\lim_{x\to\infty}\frac{x}{x-2}=1$$

$$\lim_{x\to-\infty}\frac{x}{x-2}=1$$

$\boxed{\text{HORIZONTAL ASYMPTOTE @ } y=1}$

XRONOS HW5 #4:

$$\lim_{h\to0}\frac{-9\sqrt{h+4}+18}{h}=\lim_{h\to0}\left(\frac{-9\sqrt{h+4}+18}{h}\right)\left(\frac{-9\sqrt{h+4}-18}{-9\sqrt{h+4}-18}\right)$$

$$=\lim_{h\to0}\frac{81(h+4)-324}{h(-9\sqrt{h+4}-18)}$$

$$=\lim_{h\to0}\frac{81h+324-324}{h(-9\sqrt{h+4}-18)}$$

$$= \lim_{h \to 0} \frac{81K}{K(-9\sqrt{h+4} - 18)} = \frac{81}{-9\sqrt{0+4} - 18}$$

$$= \frac{81}{-9(2)-18} = \frac{81}{-36} = \boxed{-\frac{9}{4}}$$

XRONOS HW5 #5:

THE POSITION FUNCTION IS GIVEN BY $s(t) = -16t^2 + 23t + 1$, WHERE $s(t)$ IS THE HEIGHT ABOVE THE GROUND AFTER t SECONDS. FIND THE AVERAGE VELOCITY OF THE BALL STARTING WITH $t=4$ TO THE TIME 0.5 SECONDS LATER.

NOTE: 0.5 SECONDS LATER IS 4.5 SECONDS

AVERAGE VELOCITY ON [4, 4.5]: $\frac{s(4.5) - s(4)}{4.5 - 4}$

$$\Rightarrow \frac{(-16(4.5)^2 + 23(4.5) + 1) - (-16(4)^2 + 23(4) + 1)}{0.5}$$

$$= \frac{-324 + 103.5 + 1 + 256 - 92 - 1}{0.5} = -113$$

NOW, FIND THE AVERAGE VELOCITY ON [4, 4+h], $h \neq 0$

$$v = \frac{s(4+h) - s(4)}{4+h-4} = \frac{(-16(4+h)^2 + 23(4+h) + 1) - (-16(4)^2 + 23(4) + 1)}{h}$$

$$= \frac{-16(16 + 8h + h^2) + 92 + 23h + 1 + 163}{h}$$

$$= \frac{-256 - 128h - 16h^2 + 92 + 23h + 1 + 163}{h}$$

$$= \frac{-16h^2 - 105h}{h} = \frac{\cancel{h}(-16h-105)}{\cancel{h}} = \boxed{-16h-105}$$

$$\lim_{h\to 0} \frac{16(h+4)^2 - 23h - 256}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{256} + 128h + 16h^2 - 23h - \cancel{256}}{h}$$

$$= \lim_{h\to 0} - \frac{105h + 16h^2}{h} = \lim_{h\to 0} - \frac{\cancel{h}(105+16h)}{\cancel{h}}$$

$$= -105 - 16(0) = \boxed{-105}$$

XRONOS HW5 #6:

FIND THE AVERAGE VELOCITY OF $s(t) = -10t^2 + 8t + 4$ FROM $t=4$ TO $t = 4+h$

$$v = \frac{s(4+h) - s(4)}{4+h-4}$$

$$v = \frac{(-10(4+h)^2 + 8(4+h) + 4) - (-10(4)^2 + 8(4) + 4)}{h}$$

$$v = \frac{(-10(16+8h+h^2) + 32 + 8h + 4) - (-160 + 32 + 4)}{h}$$

$$= \frac{-160 - 80h - 10h^2 + 32 + 8h + 4 + 124}{h}$$

$$= \frac{-72h - 10h^2}{h} = \frac{\cancel{h}(-72-10h)}{\cancel{h}} = \boxed{-72-10h}$$

XRONOS HW5 #3:

$$\lim_{x\to -1} \frac{-\frac{5}{x} - 5}{x+1} = \lim_{x\to -1} \left(\frac{-\frac{5}{x} - 5}{x+1}\right)\left(\frac{x}{x}\right)$$

$$= \lim_{x\to -1} \frac{-5-5x}{x^2+x} = \lim_{x\to -1} \frac{-5(\cancel{x+1})}{x(\cancel{x+1})} = \frac{-5}{-1} = \boxed{5}$$

XRONOS HW5 #1:

$$\lim_{h\to 0} \frac{(h-3)^2-9}{h} = \lim_{h\to 0} \frac{h^2-6h+9-9}{h} = \lim_{h\to 0} \frac{h^2-6h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(h-6)}{\cancel{h}} = 0-6 = \boxed{-6}$$

WHAT FUNCTION IS BEING DIFFERENTIATED?

$$\lim_{h\to 0} \frac{\boxed{(h-3)^2}-9}{h}$$

$f(x+h)=(h-3)^2 \Rightarrow x=0\,!$

$f(x)=9$

$\boxed{f(x)=(x-3)^2}$, $x=0$

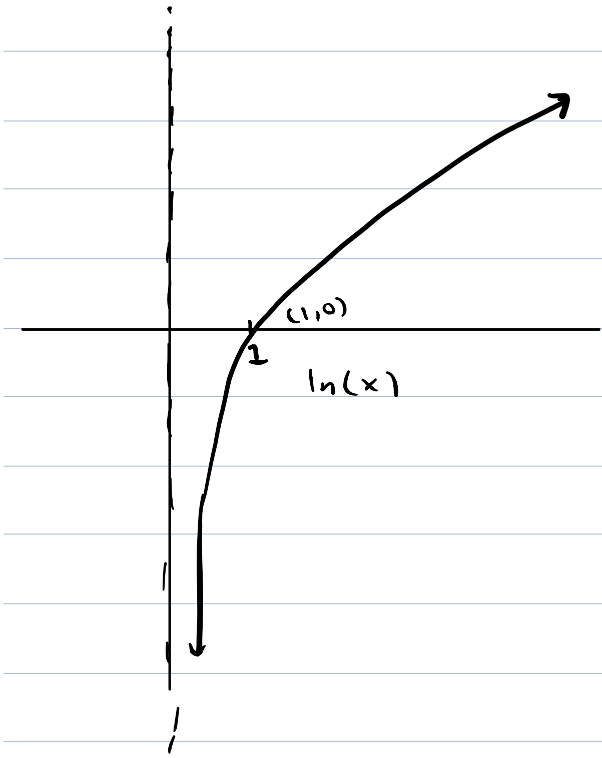
$\Rightarrow f(x+h)=(x+h-3)^2$

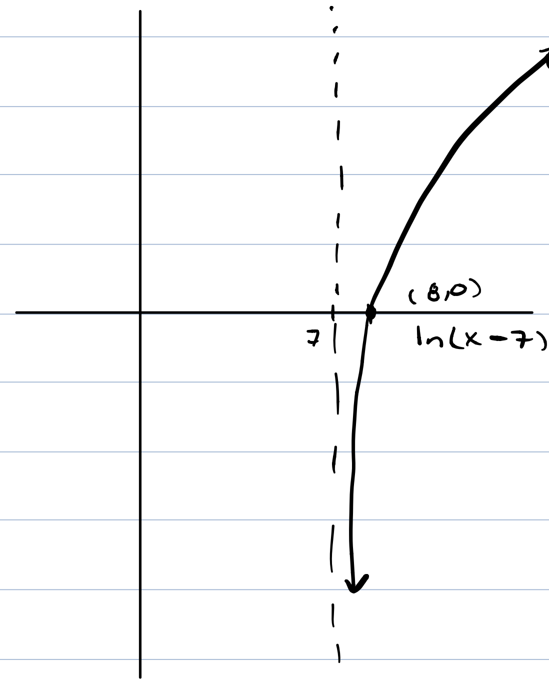
$=(h-3)^2$

$f(0)=(0-3)^2=9$ ✓

AT WHAT X-VALUE ARE YOU COMPUTING THE DERIVATIVE?

$\boxed{x=0}$


(1,0)
1
ln(x)


(8,0)
7
ln(x-7)