XRONOS HW 16 #2:

FIND CRITICAL NUMBERS: $f(x) = \frac{1}{5}x^5 + \frac{3}{4}x^4 + x^3 + \frac{1}{2}x^2 + 6$

$f'(x) = x^4 + 3x^3 + 3x^2 + x$

SET $f'(x)=0$ AND SOLVE:

$f'(x)=0 \Rightarrow x^4+3x^3+3x^2+x=0 \Rightarrow x(x^3+3x^2+3x+1)=0$

$\Rightarrow x=0$ OR $3x^3+3x^2+x+1=0$

↑ POSSIBLE RATIONAL ZEROS ARE $\pm 1, \pm \frac{1}{3}$

TRY $x=-1$: $3(-1)^3+3(-1)^2-1+1 = -3+3-1+1 = 0$ ✓

$$\begin{array}{r|rrrr} -1 & 1 & 3 & 3 & 1 \\ & \downarrow & -1 & -2 & -1 \\ \hline & 1 & 2 & 1 & 0 \\ & x^2 & x & c & r \end{array}$$

$\Rightarrow x^3+3x^2+3x+1 = (x+1)(x^2+2x+1) = (x+1)(x+1)^2$

$\Rightarrow x=-1$, MULTIPLICITY 3

CRITICAL NUMBERS: $\boxed{x=-1,0}$

XRONOS HW 16 #9:

$f(x) = 2x^3+3x^2$, FIND ABSOLUTE MAX AND MIN ON $[1,2]$:

1. EVALUATE f AT $1,2$:

$f(1) = 2(1)+3(1) = 5$, $f(2) = 2(8)+3(4) = 28$

2. FIND CRITICAL POINTS OF f ON $(1,2)$:

$f'(x) = 6x^2+6x$

$f'(x)=0 \Rightarrow 6x^2+6x=0 \Rightarrow 6x(x+1)=0 \Rightarrow x=0,-1$

NEITHER OF THE CRITICAL POINTS LIE IN THE INTERVAL $[1,2]$, SO

$\boxed{\text{ABSOLUTE MAX: } 28, \text{ ABSOLUTE MIN: } 5}$

XRONOS HW 16 #6:

FIND CRITICAL NUMBERS: $f(x) = \cos(x) + \sin^2(x)$ ON $[0, 2\pi]$

$f'(x) = -\sin(x) + 2\sin(x)\cos(x)$

SET $f'(x) = 0$ AND SOLVE:

$f'(x) = 0 \Rightarrow -\sin(x) + 2\sin(x)\cos(x) = 0$

$\Rightarrow \sin(x)[-1 + 2\cos(x)] = 0$

$\Rightarrow \underbrace{\sin(x) = 0}$ OR $\underbrace{\cos(x) = 1/2}$

$\boxed{x = 0, \pi, 2\pi}$ $\quad \uparrow \boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

XRONOS HW 16 #5:

FIND CRITICAL NUMBERS: $f(x) = x^2\sqrt{1 - x^2}$

$f(x) = x^2(1 - x^2)^{1/2}$

$f'(x) = \left[\frac{d}{dx}x^2\right]\left[(1 - x^2)^{1/2}\right] + \left[x^2\right]\left[\frac{d}{dx}(1 - x^2)^{1/2}\right]$

$f'(x) = 2x\sqrt{1 - x^2} + x^2\left(\frac{1}{2}(1 - x^2)^{-1/2}(-2x)\right) \Rightarrow f'(x) = 2x\sqrt{1 - x^2} - \frac{x^3}{\sqrt{1 - x^2}}$

SET $f'(x) = 0$ AND SOLVE:

$f'(x) = 0 \Rightarrow 2x\sqrt{1 - x^2} - \frac{x^3}{\sqrt{1 - x^2}} = 0 \quad \Rightarrow x\left(2\sqrt{1 - x^2} - \frac{x^2}{\sqrt{1 - x^2}}\right) = 0$

So, $x = 0$ OR $2\sqrt{1 - x^2} - \frac{x^2}{\sqrt{1 - x^2}} = 0$

$2\sqrt{1 - x^2} = \frac{x^2}{\sqrt{1 - x^2}} \Rightarrow 2(\sqrt{1 - x^2})(\sqrt{1 - x^2}) = x^2$

$\Rightarrow 2(1 - x^2) = x^2 \Rightarrow 2 - 2x^2 = x^2 \Rightarrow 2 = 3x^2 \Rightarrow x^2 = \frac{2}{3} \Rightarrow x = \pm\sqrt{\frac{2}{3}}$

FINALLY, CRITICAL POINTS OCCUR WHEN $f'(x)$ DOES NOT EXIST, ie WHEN $\sqrt{1-x^2}=0$

(BECAUSE YOU CANNOT DIVIDE BY 0)

$\sqrt{1-x^2}=0 \Rightarrow 1-x^2=0 \Rightarrow 1=x^2 \Rightarrow x=\pm 1$

IN THE DOMAIN, SO BOTH ARE CRITICAL PTS

XRONOS HW 16 #7:

$f(x)=\sin(x)\cos(x)$, FIND ABSOLUTE MINIMUM AND MAXIMUM ON $[0,2\pi]$

1. EVALUATE f AT $0, 2\pi$:

$f(0)=\sin(0)\cos(0)=0$, $f(2\pi)=\sin(2\pi)\cos(2\pi)=0$

2. FIND CRITICAL POINTS OF f ON $(0,2\pi)$:

$f'(x)=\cos(x)\cos(x)+\sin(x)(-\sin(x)) = \cos^2(x)-\sin^2(x)$

$f'(x)=0 \Rightarrow \cos^2(x)-\sin^2(x)=0$

$\Rightarrow \cos^2(x)=\sin^2(x) \Rightarrow 1=\dfrac{\sin^2(x)}{\cos^2(x)} = \tan^2(x)$

$x=\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

3. EVALUATE f @ CRITICAL POINTS:

$f(\pi/4)=\sin(\pi/4)\cos(\pi/4)=(\sqrt{2}/2)(\sqrt{2}/2) = 2/4 = 1/2$

$f(3\pi/4)=-1/2$, $f(5\pi/4)=1/2$, $f(7\pi/4)=-1/2$

$\Rightarrow$ $\boxed{\begin{array}{l}\text{ABSOLUTE MAX: } 1/2 \\ \text{ABSOLUTE MIN: } -1/2\end{array}}$

XRONOS HW 16 #10:

$f(x)=xe^{-x}$, FIND ABSOLUTE MAX AND MIN ON $[0,2]$:

1. EVALUATE f AT 0, 2:

$f(0)=0$, $f(2)=2e^{-2}=2/e^2$

2. FIND CRITICAL POINTS OF f ON $(0,2)$:

$f'(x) = e^{-x} + x(e^{-x})(-1) = e^{-x} - xe^{-x}$

$f'(x)=0 \Rightarrow e^{-x} - xe^{-x} = 0 \Rightarrow e^{-x}(1-x)=0$

$\Rightarrow 1-x=0 \Rightarrow x=1$

3. EVALUATE f AT CRITICAL POINT:

$f(1) = e^{-1} = 1/e$

$\boxed{\text{ABSOLUTE MAX: } 1/e \quad \text{ABSOLUTE MIN: } 0}$ ↙ BECAUSE $1/e > 2/e^2$.

*IF TIME

XRONOS HW 16 #15: DOES THE EXTREME VALUE THEOREM HOLD FOR

$$f(x) = \begin{cases} 6(x-3)^3 - 1 & x < 4 \\ \ln(x-3) - 10 & x \geq 4 \end{cases} \quad \text{OVER } [-4,14]?$$

1. INTERVAL IS CLOSED ✓

2. IS f CONTINUOUS ON $[-4,14]$?

*CHECK "CHANGE POINT", $x=4$:

$\lim_{x\to4^+} f(x) = \lim_{x\to4^+} \ln(x-3) - 10 = \ln(4-3) - 10 = \ln(1) - 10 = 0 - 10 = -10$

$\lim_{x\to4^-} f(x) = \lim_{x\to4^-} 6(x-3)^3 - 1 = 6(4-3)^3 - 1 = 6 - 1 = 5$

$\lim_{x\to4^+} f(x) \neq \lim_{x\to4^-} f(x)$, SO f IS NOT CONTINUOUS AT $x=4$ AND $4 \in [-4,14]$, SO

THE EXTREME VALUE THEOREM DOES NOT HOLD.

XRONOS HW 16 #16: DOES THE EXTREME VALUE THEOREM HOLD FOR

$$f(x) = \begin{cases} 4e^{(x-2)} - 1, & x < 2 \\ 5e^{(x-2)} + 8, & x \geq 2 \end{cases} \quad \text{OVER } [-8,10]?$$

1. INTERVAL IS CLOSED ✓

2. IS f CONTINUOUS ON $[-8,10]$?

*CHECK "CHANGE POINT", $x=2$:

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 5e^{(x-2)}+8 = 5e^{(2-2)}+8 = 5(1)+8 = 13$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} 4e^{(x-2)}-1 = 4e^{(2-2)}-1 = 4(1)-1 = 3$

$\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$, SO f IS NOT CONTINUOUS AT $x=2$ AND $2 \in [-8,10]$, SO

THE EXTREME VALUE THEOREM DOES NOT HOLD.

XRONOS HW17#7: FIND ALL VALUES c THAT SATISFY THE CONCLUSION OF ROLLES THEOREM IF $f(x) = \tan^2(x)$ ON $[-\pi/4, \pi/4]$

FIND c SUCH THAT $f'(c) = 0$, FOR $c \in (-\pi/4, \pi/4)$

$f(c) = \tan^2(c) = \tan(c)\tan(c)$

$f'(c) = \sec^2(c)\tan(c) + \tan(c)\sec^2(c) = 2\tan(c)\sec^2(c)$

$f'(c) = 0 \Rightarrow 2\tan(c)\sec^2(c) = 0$

EITHER $\tan(c) = 0$ OR $\sec^2(c) = 0$

$\tan(c) = 0 \Rightarrow c = 0, n\pi$

$\sec^2(c) = 0 \Rightarrow$ NO SOLUTION

SO, $c = 0$ OR $n\pi$, BUT ONLY $\boxed{c=0}$ IS IN $(-\pi/4, \pi/4)$.

#IF TIME

XRONOS HW17#8: LET $f(x) = \cot(x)$ ON $[0, \pi]$. DOES $f(x)$ SATISFY THE HYPOTHESES OF THE MEAN VALUE THEOREM?

NO, $f(x)$ IS NOT CONTINUOUS AT $x=0$ OR $x=\pi$ (VERTICAL ASYMPTOTES HERE)

SO f IS NOT CONTINUOUS ON $[0, \pi]$.

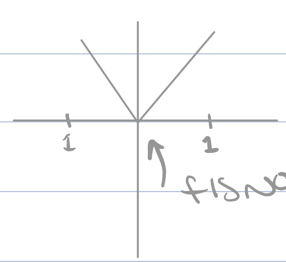
XRONOS HW17#9: LET $f(x) = \ln(x)$ ON $[1, e]$. DOES $f(x)$ SATISFY THE HYPOTHESES OF THE MEAN VALUE THEOREM?

1. f IS CONTINUOUS ON $[1, e]$ ✓

2. f IS DIFFERENTIABLE ON $(1, e)$

XRONOS HW17#1:

DOES $f(x) = |x|$ SATISFY THE THREE HYPOTHESES OF ROLLE'S THEOREM ON $[-1, 1]$?



f IS NOT DIFFERENTIABLE AT 0!

f IS NOT DIFFERENTIABLE AT 0, SO ROLLE'S THEOREM IS NOT SATISFIED.

XRONOS HW17#3:

DOES $f(x)=\frac{1}{x}$ SATISFY THE THREE HYPOTHESES OF ROLLE'S THEOREM ON $[-1,1]$?

NO, BECAUSE $f(-1)=\frac{1}{-1}=-1$ AND $f(1)=\frac{1}{1}=1$, SO $f(-1)\neq f(1)$.

XRONOS HW17#11: FIND ALL NUMBERS, c, THAT SATISFY THE CONCLUSION OF THE MEAN VALUE THEOREM FOR $f(x)=2x^2-3x+1$ ON $[0,2]$

FIND c SUCH THAT $f'(c)=\frac{f(2)-f(0)}{2-0}$ FOR $c\in(0,2)$

NOTE: $f(c)=2c^2-3c+1 \Rightarrow f'(c)=4c-3$

$f(2)=2(4)-3(2)+1=8-6+1=3$

$f(0)=2(0)-3(0)+1=1$

$\Rightarrow f'(c)=\frac{3-1}{2} \Rightarrow 4c-3=1 \Rightarrow 4c=4 \Rightarrow \boxed{c=1}$

XRONOS HW17#12: FIND ALL NUMBERS, c, THAT SATISFY THE CONCLUSION OF THE MEAN VALUE THEOREM FOR $f(x)=\sqrt{x}+2$ ON $[4,9]$

FIND c SUCH THAT $f'(c)=\frac{f(9)-f(4)}{9-4}$

NOTE: $f(c)=\sqrt{c}+2 \Rightarrow f'(c)=\frac{1}{2}c^{-1/2}=\frac{1}{2\sqrt{c}}$

$f(9)=\sqrt{9}+2=3+2=5$; $f(4)=\sqrt{4}+2=2+2=4$

$\Rightarrow f'(c)=\frac{5-4}{9-4}=\frac{1}{5} \Rightarrow \frac{1}{2\sqrt{c}}=\frac{1}{5}$

$\Rightarrow \frac{1}{2}=\sqrt{c}\cdot\frac{1}{5} \Rightarrow \frac{5}{2}=\sqrt{c} \Rightarrow \left(\frac{5}{2}\right)^2=c \Rightarrow \boxed{c=\frac{25}{4}}$

XRONOS HW17#13: IF $f(1)=10$ AND $f'(x) \geq 2$ FOR $1 \leq x \leq 4$, HOW SMALL CAN $f(4)$ POSSIBLY BE?

USE MEAN VALUE THEOREM FOR $x \in [1,4]$

$f'(x) = \frac{f(4)-f(1)}{4-1} \Rightarrow f'(x) = \frac{f(4)-10}{3} \geq 2$, SO $f(4)-10 \geq 6$

$\Rightarrow \boxed{f(4) \geq 16}$