Module 1 Lecture Notes

MAC1105

Fall 2019

1 Real and Complex Numbers

1.1 Subgroups of Real Numbers

Definition

A set is a collection of ________________ ________________. An ________________ is an object that is in a specified set. An interval is a collection of ________________ ________________.

Note 1. We can describe the elements of a set using **set builder notation**. An example of this is shown below.

Example 1. To describe the set of all freshman at the University of Florida in set builder notation, we would write:

Natural Numbers

The **natural numbers** are the numbers we use for counting:

Whole Numbers

The set of natural numbers plus zero is the set of **whole numbers**:

Integers

The set of **integers** is the set of negative natural numbers plus the whole numbers:

____

Rational Numbers

The set of **rational numbers** includes fractions written as $\frac{m}{n}$, where m, n are ________________ and $n \neq 0$:

____

Irrational Numbers

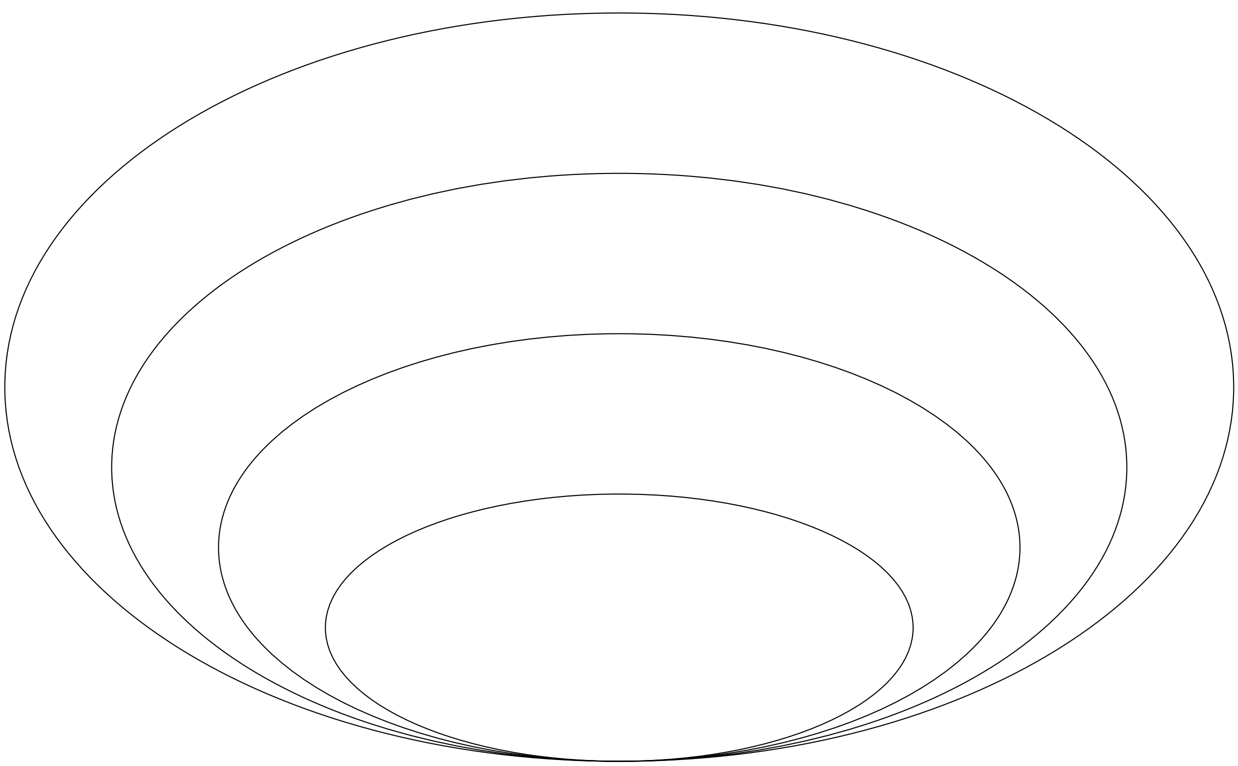
The set of **irrational numbers** is the set of numbers that are not ____________________, are non-____________________, and are non-____________________.

Real Numbers

The set of **real numbers** is the set of rational and irrational numbers together:

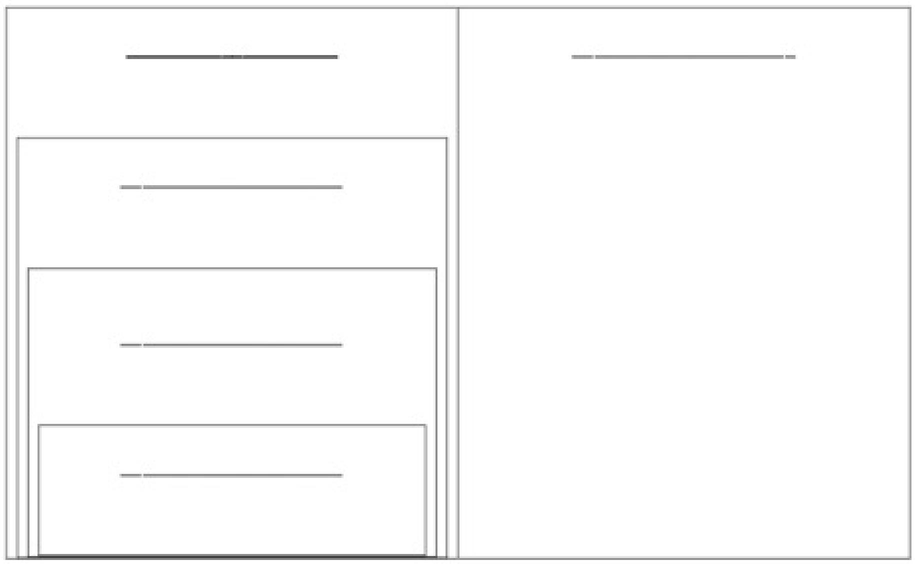
Definition

Let A and B be sets. Say that A is a ____________________ of B, written $A \subseteq B$, if every element of A is an element of B.



Example 2. Classify the following numbers in the chart provided:

$$-\frac{21}{7}, \frac{21}{7}, \frac{\pi}{4}, \frac{4}{\pi}, \frac{0}{\pi}, \frac{\pi}{0}, \sqrt{9}, \sqrt{-9}, \sqrt{2}, \sqrt{-2}$$



Example 3. Which of the following is the smallest set of real numbers that $\sqrt{\frac{0}{36}}$ belongs to?

1. Natural Number
2. Whole Number
3. Integer
4. Rational
5. Irrational
6. Not a Real Number

Example 4. Which of the following is the smallest set of real numbers that 1.324324324324324... belongs to?

1. Natural Number
2. Whole Number
3. Integer
4. Rational
5. Irrational
6. Not a Real Number

1.2 Subgroups of Complex Numbers

Definition

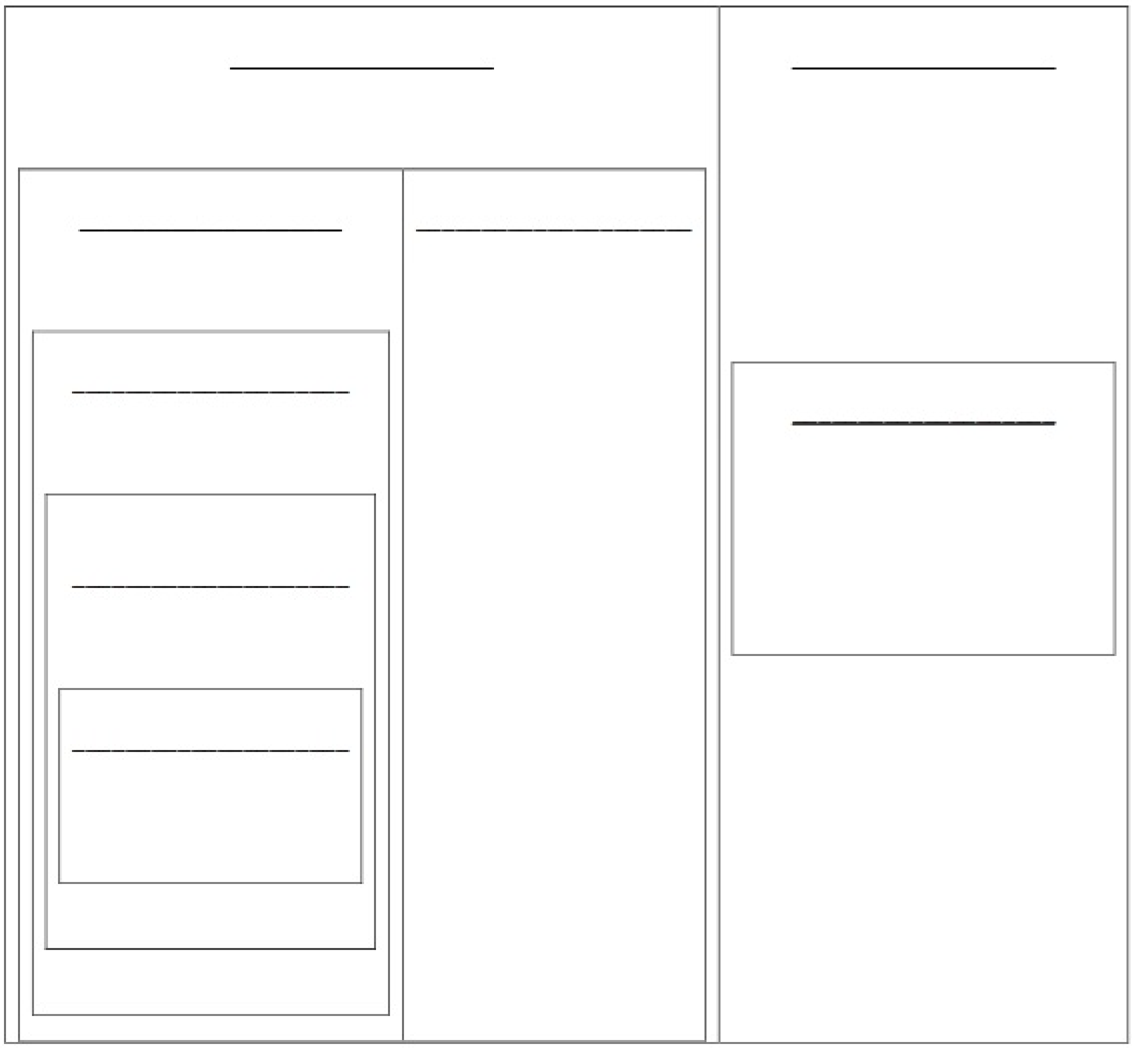
A complex number is a number of the form ______________________, where a is the real part of the complex number and b is the ______________________ part of the complex number.

Definition

Let $a+bi$ be a complex number. If $a = 0$ and if $b \neq 0$, then $a+bi$ is called a ______________________ ______________________ number. An imaginary number is an even root of a negative integer.

Example 5. Classify the following numbers in the chart provided:

$$-\frac{27}{9}+i, \sqrt{-4}, \sqrt{-21}, \sqrt{9}, \sqrt{11}, \frac{\pi}{0}i, \frac{8}{\pi}i, -\frac{24}{6}, -\frac{8}{3}, \frac{0}{\pi}$$



1.3 Order of Operations

Properties of Real Numbers

Let a, b, and c be real numbers.

The Inverse Properties:

There is a unique number 0, called the **additive identity** such that:

There is a unique number 1, called the **multiplicative identity** such that:

The Identity Properties:

There is a unique number $-a$, called the **additive inverse** or **negative** of a such that:

If $a \neq 0$, there is a unique number $\frac{1}{a}$, called the **multiplicative inverse** or **reciprocal** of a such that:

The Closure Property:

The Commutative Properties:

The Associative Properties:

The Distributive Property:

Definition

Operations in mathematics must be performed in a systematic order, which can be remembered by the acronym PEMDAS:

P - ____________________

E - ____________________

M - ____________________

D - ____________________

A - ____________________

S - ____________________

Note 2. In other words, to simplify mathematical expressions, we will:

1. Simplify any expressions within grouping symbols, such as () and []

2. Simplify any expressions containing ____________ or ____________

3. Perform ____________ and division IN THE ORDER IN WHICH THEY APPEAR from left to right

4. Perform addition and ____________ IN THE ORDER IN WHICH THEY APPEAR from left to right

Example 6. Use the order of operations (PEMDAS) to simplify the following expressions:

1. $12 - 10 \div (2 * 5)$

2. $7 + 10^2 \div (2 * 5) + 12$

3. $3(2)^2 - 4(6 + 2)$

1.4 Operate on Complex Numbers

Note 3. $\sqrt{-a}$ = =

Adding and Subtracting Complex Numbers

Adding Complex Numbers:

$$(a + bi) + (c + di) = \underline{\hspace{3cm}} + \underline{\hspace{3cm}}\, i$$

Subtracting Complex Numbers:

$$(a + bi) - (c + di) = \underline{\hspace{3cm}} + \underline{\hspace{3cm}}\, i$$

Multiplying Complex Numbers

Multiplying Complex Numbers by Real Numbers:

$$k(a + bi) = \underline{\hspace{2cm}} + \underline{\hspace{2cm}}\, i$$

Multiplying Complex Numbers by Complex Numbers:

Example 7. Multiply:

$$(3-4i)(2+3i)$$

Example 8. Multiply:

$$(2+3i)(4-i)$$

Definition

The complex conjugate of a complex number, $(a+bi)$ is ______________. In other words, the complex conjugate of a complex number is found by changing the sign of the imaginary part of the complex number.

Note 4. The product of $a+bi$ with its complex conjugate, $a-bi$ is $(a+bi)(a-bi)=a^2+b^2$.

Dividing Complex Numbers

To divide $a + bi$ by $c + di$, where c and d are both nonzero, multiply the fraction by the complex conjugate of $c + di$:

Example 9. Divide : $\dfrac{2+5i}{4-i}$