Module 10L Lecture Notes

MAC1105

Summer B 2019

10 Synthetic Division

10.1 Divide With Synthetic Division

The Division Algorithm

The Division Algorithm states that, given a polynomial dividend, $f(x)$, and a nonzero polynomial divisor, $d(x)$, where the degree of $d(x)$ is less than or equal to the degree of $f(x)$, there exist unique polynomials $q(x)$ and $r(x)$ such that

$$\rule{4cm}{0.4pt}$$

$q(x)$ is the ____________________ and $r(x)$ is the ____________________. The remainder is either 0 or has degree strictly less than ____________________. If $r(x) = 0$, then $d(x)$ ____________________ ____________________ into $f(x)$. This means that $d(x)$ and $q(x)$ are ____________________ of $f(x)$.

How to use Long Division to Divide a Polynomial by a Binomial

1. Set up the division problem.

2. Determine the first term of the quotient by dividing the leading term of the

____________________ by the leading term of the ____________________.

3. Multiply the answer by the divisor and write it below the ____________________ ____________________ of the dividend.

4. Subtract the bottom ____________________ from the top binomial.

5. Bring down the next term of the dividend.

6. Repeat steps 2-5 until you reach the last term of the dividend.

7. If the remainder is non-zero, express the answer using the divisor as the ____________________.

Example 1. Use long division to divide $4x^3 + 12x^2 - 24x - 28$ by $x + 4$

Definition

____________________ ____________________ is a shortcut that can be used when the divisor is a binomial in the form $x - k$, where k is a real number. In synthetic division, only the ____________________ are used in the division process.

Use synthetic Division to Divide Two Polynomials

1. Write k for the ____________________.
2. Write the coefficients of the dividend.
3. Bring down the ____________________ ____________________.
4. Multiply the leading coefficient by k. Write the product in the next column.
5. Add the terms of the second column.
6. Multiply the result by k. Write the product in the next column.
7. Repeat steps 5 and 6 for the remaining columns.
8. Use the bottom numbers to write the quotient. The number in the last column is the ____________________ and it has degree 0, the next number from the right has degree 1, the next number from the right has degree 2, etc.

Example 2. Use synthetic division to divide $6x^3 - 18x^2 + 19$ by $x - 2$.

Example 3. Use synthetic division to divide $16x^3 + 8x^2 - 32x - 20$ by $4x + 4$.

10.2 Possible Rational Roots

Rational Root Theorem

The possible rational roots of the polynomial

$$a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$$

are of the form $\pm\frac{p}{q}$, where p is a divisor of __________ and q is a divisor of __________.

How to Use the Rational Root Theorem to Find the Zeros of a Polynomial $f(x)$

1. Determine all divisors of the constant term a_0 and all divisors of the leading coefficient a_n.
2. Use 1 to determine all possible values of $\pm\frac{p}{q}$, where p is a divisor of __________ and q is a divisor of __________.
3. Determine which possible zeros are actually zeros of $f(x)$ by evaluating each case of $f\left(\pm\frac{p}{q}\right)$.

Example 4. Find the possible rational roots of the following polynomial:

$$f(x) = 6x^3 - 17x^2 + 6x + 8$$

Example 5. Find the possible rational roots of the following polynomial and then find the actual roots by factoring or using the Quadratic Formula:

$$f(x) = x^2 - 15$$

10.3 Completely Factor Polynomials

The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$ then the value of __________ is the remainder.

The Factor Theorem

k is a zero of $f(x)$ if and only if ____________________ is a factor of $f(x)$.

How to Find the Zeros of a Polynomial $f(x)$ Using Synthetic Division

1. Use the ____________________ ____________________ ____________________ to find all of the possible rational roots (zeros) of $f(x)$.
2. Use synthetic division to evaluate a given possible zero. If the remainder is 0, the candidate is a zero. If the remainder is not 0, discard the candidate.
3. Repeat step 2 using the quotient found with synthetic division. Continue (if possible) until the quotient is a quadratic.
4. Find the zeros of the quadratic.

Example 6. Factor the polynomial below and list all of the actual zeros for the polynomial:

$$f(x) = x^3 - 6x^2 - 15x + 100$$

Example 7. Factor the polynomial below and list all of the actual zeros for the polynomial:

$$f(x) = x^4 + 12x^3 + 37x^2 - 30x - 200$$

Example 8. Factor the polynomial below and list all of the actual zeros for the polynomial:

$$f(x) = 3x^3 + 7x^2 - 11x - 15$$

Example 9. Factor the polynomial below and list all of the actual zeros for the polynomial:

$$f(x) = 18x^3 - 9x^2 - 38x + 24$$