Module 2 Lecture Notes

MAC1105

Fall 2019

2 Linear Functions

2.1 Construct a Linear Function From Points

Definition

An ______________________ in x is a statement that two algebraic expressions are equal.

Definition

A linear equation is the equation of a straight line written in one variable. A linear equation (in one variable) can be written in the form ______________________ where $a \neq 0$.

Note 1. The only power of the variable in a linear equation is 1. For example, $x^2 - 4 = 0$ is not a linear equation because the power of the variable x is 2.

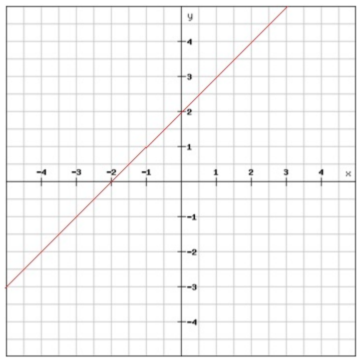
Definition

The ______________________ of a line is used to find the steepness and direction of a line. The vertical change between two points is called the ______________________, and the horizontal change between two points is called the ______________________. So, slope $= \frac{\text{rise}}{\text{run}}$. Given two pints on a line, (x_1, y_1) and (x_2, y_2), the slope of the line between the two points is given by

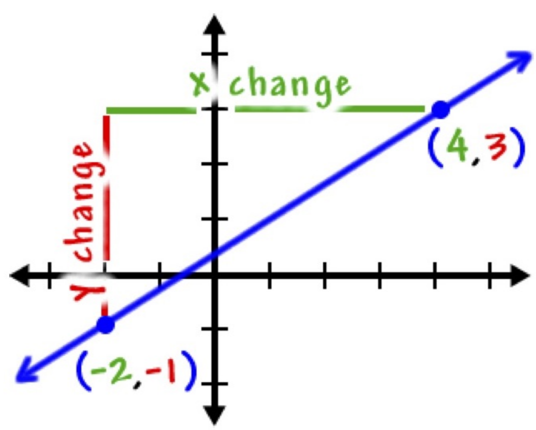
$$m = \underline{\hspace{3cm}}$$

where m denotes the slope.

Note 2. On a graph, the slope can be represented as:



Example 1. What is the slope of the line below?



Note 3. It does not matter which point you choose as (x_1, y_1) and which you choose as (x_2, y_2). But, make sure you stay consistent with the order of the y terms and the order of the x terms in

the numerator and denominator.

Definition

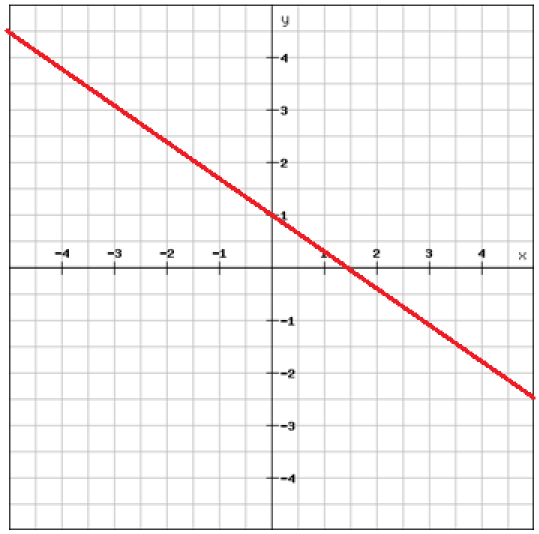
The ______ - ______________________ of a line is the point at which the line crosses the y-axis. This is the point where $x = 0$.

Definition

The ______ - ______________________ of a line is the point at which the line crosses the x-axis. This is the point where $y = 0$.

Note 4. The slope of a line can be positive, negative, undefined, or 0:

Example 2. Identify the slope, y-intercept, and x-intercept of the line below.



Slope-Intercept Form

The slope-intercept form of an equation is $y = mx + b$, where m is the ____________________ and b is the _____ - ____________________

Note 5. If we are given an equation that is not in slope-intercept form, we can still always find the y-intercept by plugging in $x = 0$ and solving for y.

Example 3. Find the equation of the line containing the two points:

$$(2, -1), \ \ (3, 4)$$

Example 4. Find the equation of the line containing the two points:

$$(-2, 3), \ \ (6, -5)$$

Point-Slope Form

Given the slope and one point on a line, we can find the equation of the line using point-slope form:

$$\rule{4cm}{0.4pt}$$

where m is the ________________ and (x_1, y_1) is the point that we are given.

Definitions

- Two lines are ________________ if they have the same slope but have different y-intercepts.
- Two lines are ________________ if the slope of one line is the negative reciprocal of the other.

Note 6. To find the negative reciprocal of a number, put the number in fraction form and switch the numerator and denominator, then negate the number. For example,

How to Find the Equation of Parallel and Perpendicular Lines:

1. Determine the slope of the given line. I suggest writing the equation in slope-intercept form first.
2. If you are finding a parallel line, the slope is the same as the original line. If you are finding a perpendicular line, the slope is the ____________________ ____________________ of the original line.
3. Use the point you are given and the slope you found in step 2 to determine the equation of the line.

Example 5. Write the equation of a line that is parallel to $y = 6x + 1$ and passes through the point $(-7, 1)$.

Example 6. Write the equation of a line that is perpendicular to $x + 3y = 6$ and passes through the point $(1, 5)$.

2.2 Converting Between Linear Forms

The Standard Form of a Line

The standard form of a line is ____________________ where A, B, and C are integers.

Note 7. When the equation of a line is written in standard form, the x and y terms are on one side of the equal sign, and the constant term is on the other side of the equal sign.

Example 7. Convert the linear function below from standard form to slope-intercept form:

$$7x + 5y = 2.$$

Example 8. Convert the linear function below from slope-intercept form to standard form:

$$y = -\frac{5}{4}x - \frac{5}{2}.$$

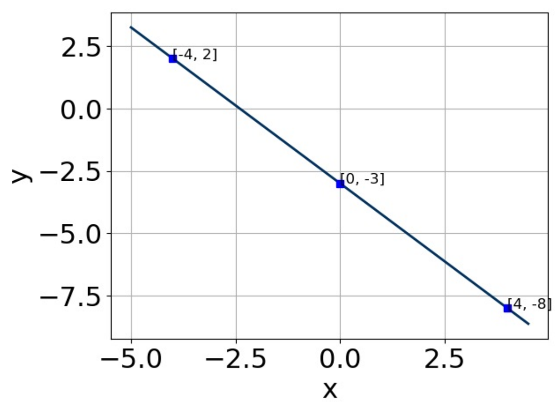
2.3 Convert Between a Linear Equation and its Graph

How to Determine the Equation of a Line Given its Graph

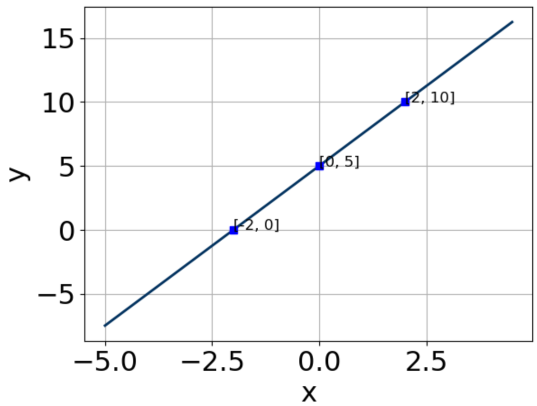
To find the equation of a line in slope-intercept form given its graph,

1. Find the slope of the line using the formula for the slope between two points.
2. Use the slope and any point on the graph to find the y-intercept, or find the y-intercept on the graph.
3. Write the equation in the form $y = mx + b$.

Example 9. Write the equation of the line below in slope-intercept form and in standard form.



Example 10. Write the equation of the line below in slope-intercept form and in standard form.



Note 8. The equation of a vertical line is given by: ________________. The equation of a horizontal line is given by: ________________.

2.4 Solve Linear Equations

Note 9. To solve an equation is to find all values of x for which the equation is true. Such values of x are ____________________ (or roots, zeros) of the equation.

How to Solve Linear Equations in One Variable The steps below do not need to be followed in any particular order (as long as you remember PEMDAS)

1. Add, subtract, multiply, or divide an equation by a number or expression. BUT, you MUST do the same thing to both sides of the equal sign. (Math is fair)
2. Apply the distributive property (you may not always need to do this): ____________________
3. Isolate the variable on one side of the equation.
4. Solve for the variable by dividing or multiplying a constant.

Example 11. Solve the following equation:

$$-15(2x + 10) = -11(-6x + 14)$$

Example 12. Solve the following equation:

$$-2(9x-4) = -15(12x+10)$$

Definition

- A ____________________ ____________________ is the quotient (or ratio) of two polynomials.
- A ____________________ ____________________ is an equation that contains at least one rational expression.

Note 10. Recall that a rational number is the ratio of two integers. For example, $\frac{3}{2}$ and $\frac{5}{6}$ are rational numbers.

Note 11. To solve a linear equation with fractions, multiply both sides by the ____________________ _________________________ _________________________ to clear the fraction.

Note 12. If you are given a rational equation in the form of a proportion:

$$\frac{a}{b} = \frac{c}{d}$$

then you can solve the equation by cross multiplication:

Example 13. Solve the equation:

$$\frac{x-3}{7} = \frac{4x+12}{7}$$

Example 14. Solve the equation:

$$\frac{-4x-5}{3} - \frac{-5x+6}{2} = \frac{3x+8}{5}$$

Example 15. Solve the equation:

$$\frac{8x-7}{4}-\frac{-7x+7}{6}=\frac{-7x+3}{3}$$