# Module 3 Lecture Notes

MAC1105

Fall 2019

## 3 Linear Inequalities

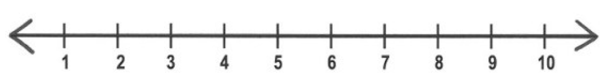
### 3.1 Set Notation

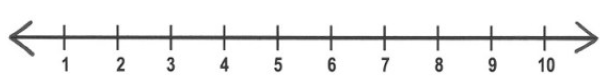
**Definition**

A set is a collection of ______________ ______________. An ______________ is an object that is in a specified set. An interval is a collection of ______________ ______________.

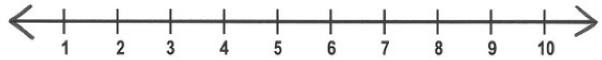
**Example 1.** The set of whole numbers is ______________.

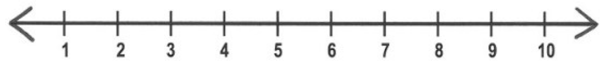
**Example 2.** (3,7) is the set of real numbers between 3 and 7 but NOT INCLUDING 3 and 7. On a number line, this looks like:



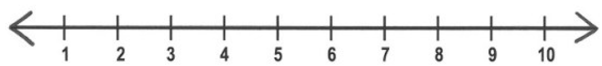




If we want to include the numbers 3 and 7 we use closed brackets, [3,7]. On a number line, this looks like:




**Note 1.** There are a couple of different ways to describe elements in a specific set. To describe solutions that exist in an interval, we can use **interval notation** ____________________. We read this as "x is an element of (a,b)", or x is some number between a and b, but is not a or b. On a number line, this looks like:




To write this in **inequality notation**, we write ____________________. We can also describe the elements of a set using **set builder notation**. An example of this is shown below.

**Example 3.** Imagine we tried to create the set of all presidents. That's a long and tedious list! But if we want to implicitly write this set, we can say $\{x : x \text{ is a president}\}$. This means __________________________________________________. If we wanted to describe the set of all students in this course (in set builder notation), we would write

__________________________________________________.

**Definition**

If $a$ and $b$ are real numbers, then an ____________________, denoted $(a, b)$ between $a$ and $b$ is the collection of all real numbers $x$ such that $a < x$ and $x < b$.

**Note 2.** Recall the inequality symbols and their meaning:

| Symbol | Meaning |
|---|---|
| | |
| | |
| | |
| | |

**Example 4.** Write the following set in interval notation and inequality notation: all real numbers between $a$ and $b$ and including $a$ and $b$:

____________________

____________________

**Example 5.** Write the following set in interval notation and inequality notation: all real numbers greater than $a$ but not including $a$:

____________________

____________________

**Example 6.** Write the following set in interval notation and inequality notation: all real numbers less than $a$ or greater than $b$:

____________________

**Example 7.** Solve the linear equation below and choose the interval that contains the solution:

$$x + 3 = 5.5$$

(a) $x = a$, where $a \in [-2, -1]$

(b) $x = a$, where $a \in [-1, 0]$

(c) $x = a$, where $a \in [0, 1]$

(d) $x = a$, where $a \in [1, 2]$

(e) $x = a$, where $a \in [2, 3]$

## 3.2 Solve Linear Inequalities

**Properties of Inequalities**

**Addition Property:** If $a < b$, then ____________________

**Multiplication Property:** If $a < b$ and $c > 0$, then _______________. If $a < b$ and $c < 0$, then _______________.

**Note 3.** The above properties also apply to $a \leq b, a > b$, and $a \geq b$.

**Example 8.** Illustrate the addition property by solving the following inequality:

$$x - 15 < 4$$

**Example 9.** Illustrate the multiplication property by solving the following inequality:

$$3x < 6$$

**Note 4.** We can solve linear inequalities similar to solving linear equations by combining like terms, performing operations, and isolating the variable on one side of the inequality.

**Note 5.** When solving an inequality, if you multiply or divide by a negative number you must FLIP THE INEQUALITY.

**Example 10.** Solve the linear inequality and write your answer in inequality notation and interval notation. It may help to graph the solution on a number line:

$$-10x - 10 \leq 9x + 8$$

**Example 11.** Solve the linear inequality and write your answer in inequality notation and interval notation. It may help to graph the solution on a number line:

$$x - \frac{9}{8} < \frac{7}{2}x - \frac{5}{3}$$

**Example 12.** Solve the linear inequality and write your answer in inequality notation and interval notation. It may help to graph the solution on a number line:

$$-x - \frac{5}{4} \geq \frac{5}{3}x + \frac{8}{7}$$

## 3.3 Solve Compound Linear Inequalities

**Definition**

A compound inequality includes __________ ____________________ in one statement.

**Example 13.** The inequality $-1 \leq x < 4$ is a compound inequality. It means that _______________

and _______________. On a number line, this looks like

**Definition**

Let $A$ and $B$ be sets.

- The ____________________ of $A$ and $B$, denoted $A \cup B$ is the set of all objects $x$ such that either $x \in A$ or $x \in B$.
- The ____________________ of $A$ and $B$, denoted $A \cap B$ is the set of all objects $x$ such that $x \in A$ and $x \in B$.

**Example 14.** Represent the following expression on the number line:

$$x < -1 \quad \text{or} \quad x > 2$$

**Note 6.** To solve compound inequalities, first split the inequality into two parts. Then, solve each inequality separately. Finally, put the two inequalities back together after solving.

**Example 15.** Solve the following inequality and write your answer in inequality notation and interval notation:

$$5x - 8 < 6x - \frac{3}{2} < 3x + 3$$

**Example 16.** Solve the following inequality and write your answer in inequality notation and interval notation:

$$-x - \frac{1}{2} < -\frac{5}{4}x + 3 \quad \text{or} \quad \frac{7}{2}x + 1 > \frac{8}{5}x - \frac{5}{6}$$

**Example 17.** Solve the following inequality and write your answer in inequality notation and interval notation:

$$-8x - 4 \leq \frac{17}{4}x + \frac{3}{8} \leq 4x + 3$$