Module 4 Lecture Notes

MAC1105

Summer B 2019

4 Quadratic Functions

4.1 Factor Trinomials

Rules for Positive Exponents

For all positive integers m and n and all real numbers a and b:

Product Rule

$a^n a^m =$

Power Rules

$(a^n)^m =$

$(ab)^{nm} =$

$\left(\frac{a}{b}\right)^n = \qquad b \neq 0$

Zero Exponent

$a^0 =$

Definition

An expression of the form $a_k x^k$ where $k \geq 0$ is an integer, a_k is a constant, and x is a variable, is called a ____________________. The constant a_k is called the ____________________ and k is the ____________________ of the monomial if $k \neq 0$.

Note 1. The sum of monomials with different degrees forms a ____________________. The monomials in the polynomial are called the ____________________. A polynomial with exactly two terms is called a ____________________ and a polynomial with exactly 3 terms is called a ____________________.

Polynomial in One Variable in Standard Form:

$$a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$$

where $a_0, ..., a_{n-1}, a_n$ are real numbers and $n \geq 0$ is an integer.

Definition

A ____________________ ____________________ is a polynomial function of degree 2.

Note 2. $x^2 + 5x + 2$ is a quadratic function, but $x + 2$ is not a quadratic function because ____.

Operations on Polynomials

Adding and Subtracting Polynomials

Polynomials are added and subtracted by combining like terms.

Multiplying Polynomials

Two polynomials are multiplied by using the properties of real numbers and the rules for exponents.

Example 1. Perform the operation:

$$(2x^4 - 3x^2 + 1)(4x - 1)$$

Note 3. When multiplying two binomials, use ____________________.

Definition

The greatest common factor (GCF) of a polynomial is the ____________________ ____________________ that divides evenly into the polynomials.

How to Factor out the Greatest Common Factor

1. Identify the GCF of the ____________________.
2. Identify the GCF of the ____________________.
3. Combine 1 and 2 to find the GCF of the expression.
4. Determine what the GCF needs to be multiplied by to obtain each term in the polynomial.
5. Write the factored polynomial as the product of the GCF and the sum of the terms we need to multiply by.

Example 2. Factor $6x^3y^3 + 45x^2y^2 + 21xy$ by factoring out the greatest common factor.

Factor a Trinomaial with Leading Coefficient 1

A trinomial of the form $x^2 + bx + c$ can be factored as $(x + p)(x + q)$, where $pq =$ _____ and $p + q =$ _____.

Note 4. Not every polynomial can be factored. Some polynomials cannot be factored, in which case we say the polynomial is prime.

How to Factor a Trinomial of the Form $x^2 + bx + c$

1. Determine all possible factors of c.
2. Using the list found in 1, find two factors p and q, in which $pq =$ _____ and $p + q =$ _____.
3. Write the factored expression as ____________________.

Note 5. The order in which you write the factored polynomial does not matter. This is because multiplication is ____________________.

Example 3. Factor the trinomial:

$$x^2 + 24x + 140$$

Factor a Trinomial by Grouping

To factor a trinomial in the form $ax^2 + bx + c$ by grouping, we find two numbers with a product of _____ and a sum of _____.

How to Factor a Trinomial of the Form $ax^2 + bx + c$ by Grouping

1. Determine all possible factors of _____.
2. Using the list found in 1, find two factors p and q, in which $pq =$ _____ and $p + q =$ _____.
3. Rewrite the original polynomial as ____________________.
4. Pull out the GCF of _______________.
5. Pull out the GCF of _______________.
6. Factor out the GCF of the expression.

Example 4. Factor the polynomial:

$$35x^2 + 48x + 16$$

Example 5. Factor the polynomial:

$$21x^2 + 40x + 16$$

Example 6. Factor the polynomial:

$$36x^2 + 19x - 6$$

Factor a Perfect Square Trinomial

A perfect square trinomial can be written as the square of a binomial:

$$\underline{\hspace{4cm}} = \underline{\hspace{4cm}}$$

or

$$\underline{\hspace{4cm}} = \underline{\hspace{4cm}}$$

How to Factor a Perfect Square Trinomial

1. Confirm that the first and last term are perfect squares.
2. Confirm that the middle term is ____________ the product of ______.
3. Write the factored form as ________________________.

Example 7. Factor the polynomial:

$$100x^2 + 60x + 9$$

Factor a Difference of Squares

A difference of squares is a perfect square subtracted from a perfect square. We can factor a difference of squares by: ________________ = ______________________.

How to Factor a Difference of Squares

1. Confirm that the first and last term are perfect squares.
2. Write the factored form as ______________________.

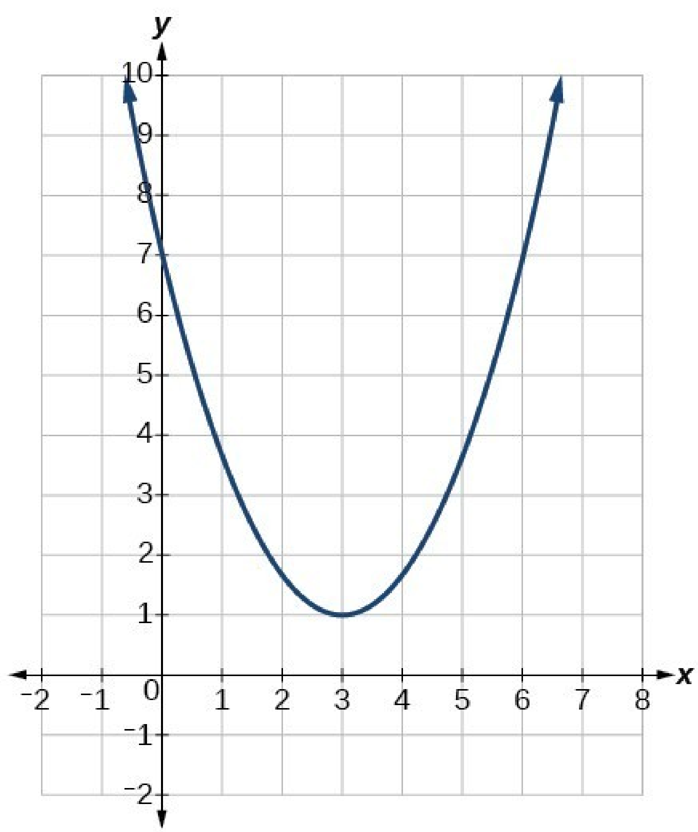
Example 8. Factor the polynomial:

$$49x^2 - 16$$

4.2 Graphing Quadratic Functions

Definitions:

The graph of a quadratic function is a U-shaped curve called a ____________________. The extreme point of a parabola is called the _______________. If the parabola opens up, the vertex represents the lowest point on the graph, called the ____________________ _______________ of the quadratic function. If the parabola opens down, the vertex represents the highest point on the graph, called the ____________________ _______________. The graph of a quadratic function is symmetric, with a vertical line drawn through the vertex called the _______________ _____ ____________________. The _____ - ____________________ are the points where the parabola crosses the x-axis.



Parabolas and Quadratic Functions

General Form of a Quadratic Function: The general form of a quadratic function is ________________________, where a, b, c are real numbers and $a \neq 0$. If ____________, the parabola opens up. If ____________, the parabola opens down.

Standard Form of a Quadratic Function: The standard form of a quadratic function is ________________________, where $a \neq 0$. If ____________, the parabola opens up. If ____________, the parabola opens down.

Axis of Symmetry: The equation to find the axis of symmetry is given by

$$x = \qquad \underline{\hspace{2cm}}.$$

Vertex: The vertex is located at (h, k), where

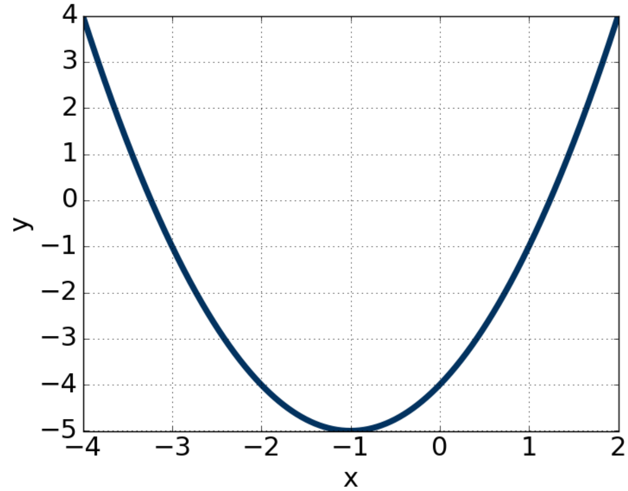
$$h = \qquad \underline{\hspace{2cm}}, \qquad \text{and} \qquad k = f(h) = f\left(\qquad \underline{\hspace{2cm}} \right)$$

How to Find the Equation of a Quadratic Function Given its Graph

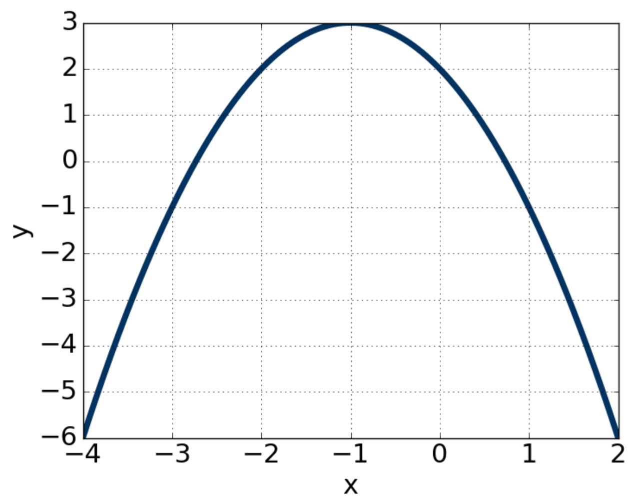
1. Identify the coordinates of the vertex, (h, k).
2. Substitute the values of h and k (found in 1) into the equation $f(x) = a(x - h)^2 + k$.
3. Substitute the values of any other point on the parabola (other than the vertex) for x and $f(x)$.
4. Solve for the stretch factor, $|a|$.
5. Determine if a is positive or negative.

6. Expand and simplify to write in general form.

Example 9. Write the equation of the graph below in the form $ax^2 + bx + c$, assuming $a = 1$ or $a = -1$:



Example 10. Write the equation of the graph below in the form $ax^2 + bx + c$, assuming $a = 1$ or $a = -1$:



Example 11. Graph the equation $f(x) = (x-3)^2 - 19$

4.3 Solving Quadratics by Factoring

How to Find the x-Intercept and y-Intercept of a Quadratic Function:

1. To find the y-intercept, evaluate the function at __________.

2. To find the x-intercepts, solve the quadratic equation ______________.

Note 6. Solving the quadratic equation $f(x) = 0$ can be done by factoring, or by using the quadratic formula. First, we will solve quadratic equations by factoring. To solve $f(x) = 0$, we will factor $f(x)$ and set each factor equal to 0.

Example 12. Solve the quadratic equation by factoring:

$$15x^2 + 9x - 6 = 0$$

Example 13. Solve the quadratic equation by factoring:

$$4x^2 + 12x + 9 = 0$$

Example 14. Solve the quadratic equation by factoring:

$$350x^2 + 30x - 8 = 0$$

4.4 Solving Quadratics using the Quadratic Formula

The Quadratic Formula:

To solve the quadratic function $f(x) = 0$, we can use the quadratic formula which is given by:

$$x = \underline{\hspace{4cm}}$$

Note 7. Recall that to find the x-intercepts of a quadratic function, we solve the quadratic equation $f(x) = 0$. So, to find the x-intercepts, we can solve by factoring, or we can solve using the quadratic formula. The quadratic formula will always work, but sometimes it is much more tedious to use.

Example 15. Solve the quadratic equation using the quadratic formula:

$$4x^2 - 8x - 8 = 0$$

Example 16. Solve the quadratic equation using the quadratic formula:

$$2x^2 - 8x + 7 = 0$$