Module 7 Lecture Notes

MAC1105

Fall 2019

7 Rational Functions

7.1 Domain of Rational Functions

Definition

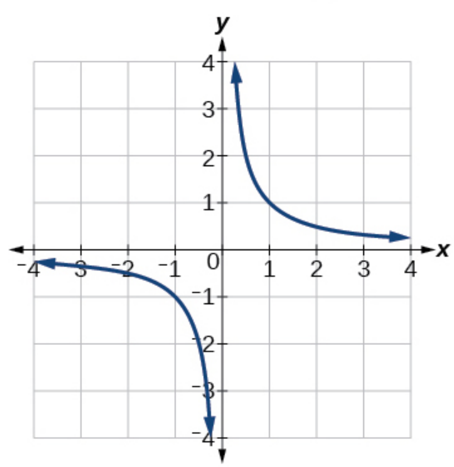
A ______________________ ______________________ is a vertical line $x = a$ where the graph tends toward positive or negative infinity as the input approaches a. We write

$$\text{As} \quad x \to a, \underline{\hspace{4cm}}$$

or

$$\text{As} \quad x \to a, \underline{\hspace{4cm}}$$

Example 1. Consider the graph of the function $f(x) = \dfrac{1}{x}$:



We cannot divide by zero, so as $x \to 0^-$, ______________________. That is, as x approaches zero from the left, $f(x)$ approaches $-\infty$. Similarly, as $x \to 0^+$, ______________________. That is, as x approaches zero from the right, $f(x)$ approaches ∞. So, there is a vertical asymptote at ______________________.

Note 1. A vertical asymptote represents a value at which a rational function is ______________________, so the value is not in the domain of the function. In general, to find the domain of a rational function we need to find the values which cause the ______________________ to be equal to zero.

How to Find the Domain of a Rational Function

1. Set the ______________________ equal to zero.
2. Solve to find the x-values that make the ______________________ equal to zero.
3. The domain is ___________ ________________ ______________________ except those found in step 2.

Example 2. Find the domain of the rational function:

$$f(x) = \frac{1}{x+3}$$

Note 2. It does not matter what type of real number makes the denominator equal to zero. When we have two or more fractions in the function, the domain is the ______________________ of the domains of each function separately. In other words, we will remove EVERY value of the function that makes the denominator equal to zero from the domain of the function.

Example 3. Find the domain of the following function:

$$f(x) = \frac{1}{3x+2} - \frac{1}{-3x-5}$$

Note 3. If we are given a rational function which has a denominator with degree larger than 1, we will first need to ____________________ the denominator and then solve for the x-values that make the denominator equal to ____________________.

Example 4. Find the domain of the rational function:

$$f(x) = \frac{1}{x^2 + 4x + 3}$$

Example 5. Find the domain of the rational function:

$$f(x) = \frac{1}{2x^3 - 3x^2 - 9x}$$

Example 6. Find the domain of the rational function:

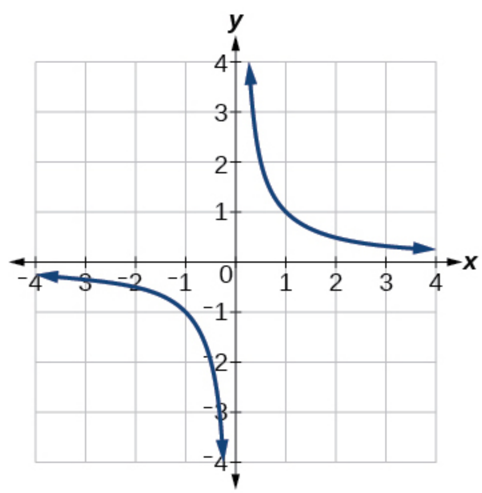
$$f(x) = \frac{1}{x^3 + 13x}$$

Definition

A ____________________ ____________________ is a horizontal line $y = b$ where the graph approaches the line as the inputs increase or decrease without bound. We write

$$\text{As} \quad x \to \infty \quad \text{or} \quad x \to -\infty, \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Example 7. Recall the graph of $f(x) = \dfrac{1}{x}$:



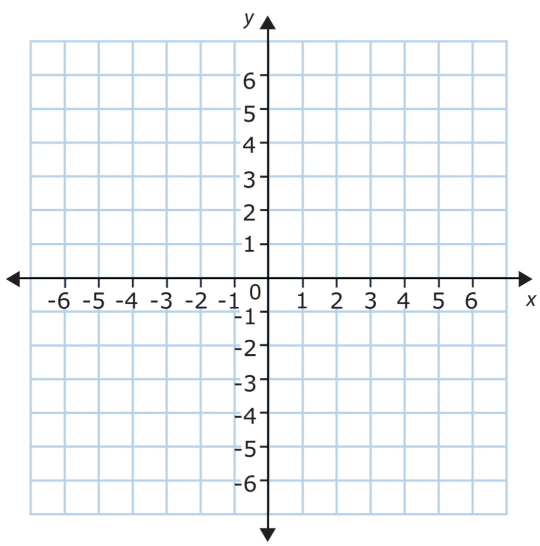
As $x \to \infty$, ______________________. That is, as x approaches infinity, $f(x)$ approaches zero.

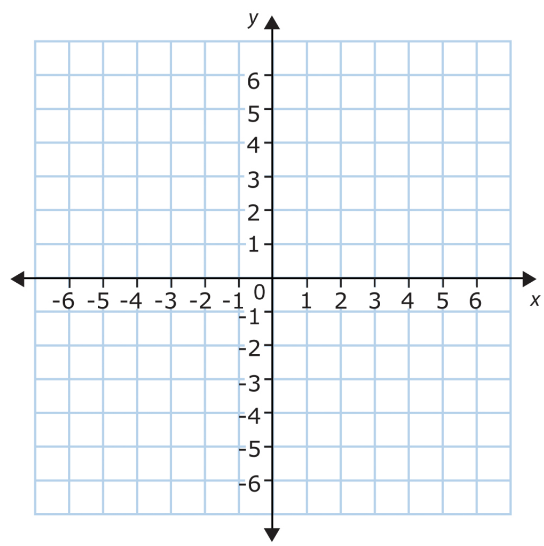
Similarly, as $x \to -\infty$, ______________________. That is, as x approaches ∞, $f(x)$ approaches zero.

So, there is a horizontal asympotote at ______________________.

7.2 Graphing Rational Functions

Graphs of Basic Rational Functions To graph rational functions in this section, we will work with two basic rational functions: $f(x) = \dfrac{1}{x}$ and $f(x) = \dfrac{1}{x^2}$.





Graphing Rational Functions With Translations

Consider the function $f(x) = \dfrac{a}{x-h} + k$.

Horizontal Shifts

Vertical Shifts

Stretch Factor

Note 4. So, we can now use the graph of $\frac{1}{x}$ to graph $f(x) = \frac{a}{x-h} + k$. When translating the graph, we must remember to shift horizontal and/or vertical asymptotes.

Note 5. To find the value of a, plug in a point on the graph after finding the value of h and k.

Graphing Rational Functions With Translations

Consider the function $f(x) = \dfrac{a}{(x-h)^2} + k$.

Horizontal Shifts

Vertical Shifts

Stretch Factor

Standard Form of Rational Functions

The standard form of $f(x) = \dfrac{a}{x-h} + k$ is

$$f(x) = \underline{\hspace{3cm}} + $$

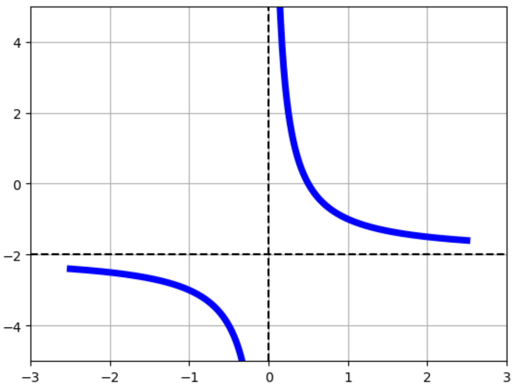
The standard form of $f(x) = \dfrac{a}{(x-h)^2} + k$ is

$$f(x) = \underline{\hspace{3cm}} + $$

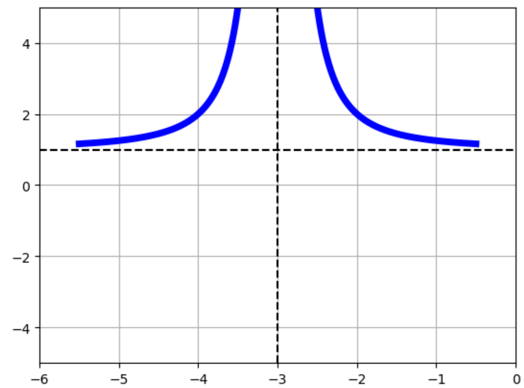
Intercepts of Rational Functions

If a rational function does not have a vertical asymptote at $x = 0$ (that is, if our function is defined at $x = 0$), then the y-intercept of the function occurs when ____________________. If a rational function does not have a horizontal asymptote at $y = 0$ (that is if our function is defined for all x-values where $f(x) = 0$), then the x-intercept of the function occurs when ____________________.

Example 8. Write an equation for the graph of the rational function below. Assume $a = 1$ or $a = -1$:



Example 9. Write an equation for the graph of the rational function below:



7.3 Solving Rational Equations

How to Solve Rational Equations

1. Identify the ____________________ (ie find the values of x for which the ____________________ are equal to 0).

2. Find the least common denominator.

3. Multiply both sides by the ____________________ ____________________ ____________________.

4. You should now have a linear or quadratic equation. Simplify your remaining equation and solve.

5. Check your solutions and check that your solutions are in the ____________________.

Example 10. Solve the rational equation below:

$$\frac{5}{x} - \frac{1}{3} = \frac{1}{x}$$

Example 11. Solve the rational equation below:

$$-\frac{8}{8x-5}+1=\frac{32}{-32x+20}$$

Example 12. Solve the rational equation below:

$$-\frac{2x^2}{3(2x^2+x-10)}-\frac{4x}{3(x-2)}=-\frac{3}{2x+5}$$

Example 13. Solve the rational equation below:

$$\frac{10}{x(x-2)} + \frac{4}{x} = \frac{5}{(x-2)}$$