Module 9L Lecture Notes

MAC1105

Summer B 2019

9 Operations on Functions

9.1 Domain

Operations on Functions

For two functions $f(x)$ and $g(x)$ with real number outputs, we define new functions $f+g$, $f-g$, fg, and $\frac{f}{g}$ by:

- $(f+g)(x) =$ ____________________
- $(f-g)(x) =$ ____________________
- $(fg)(x) =$ ____________________
- $\left(\frac{f}{g}\right)(x) =$ ______ where $g(x) \neq 0$

Example 1. Find and simplify the functions $(g-f)(x)$ and $\left(\frac{g}{f}\right)(x)$, given $f(x) = x - 1$ and $g(x) = x^2 - 1$.

Domain of Algebra of Functions

Let f and g be two functions with domains A and B. Then,

Name	Definition	Domain
$f \pm g$		
fg		
$\frac{f}{g}$		

Example 2. Determine the domain of each function below and then determine the domain of $f+g$, fg, and $\frac{f}{g}$:

$$f(x) = -5x^2 - 6x + 3$$

$$g(x) = \sqrt{3x-4}$$

Example 3. Determine the domain of each function below and then determine the domain of $f+g$, fg, and $\frac{f}{g}$:

$$f(x) = -5x^3 + 5x^2 - 5x - 6$$

$$g(x) = \sqrt{4x - 6}$$

Example 4. Determine the domain of each function below and then determine the domain of $f+g$, fg, and $\frac{f}{g}$:

$$f(x) = 6x^3 + 6x^2 - 3x + 6$$

$$g(x) = -\frac{1}{4x+3}$$

9.2 Composition

Definition

The process of combining functions so that the output of one function becomes the input of another is known as ____________ ______ ____________. For any input x and functions f and g, this action defines a ____________ ____________, which we write as $f \circ g$ such that

____

The domain of the composite function $f \circ g$ is all x such that x is in the domain of ______ and $g(x)$ is in the domain of ______.

Note 1. It is important to keep in mind the order of operations when composing functions. That is, $(f \circ g)(x) = f(g(x))$ means that the function f takes ______ as an input and yields an output of ____________.

Example 5. For the two functions below, evaluate $(f \circ g)(-3)$:

$$f(x) = 4x^2 - 4x + 4$$

$$g(x) = -3x^2 - 3x + 4$$

Example 6. For the two functions below, evaluate $(f \circ g)(3)$ and $(g \circ f)(3)$:

$$f(x) = 5x^2 - 5x - 3$$

$$g(x) = \sqrt{5x - 4}$$

Note 2. The example above shows that function compositions is not ____________________.

That is, $(f \circ g)(x) \neq$ ____________________.

Note 3. The product of functions fg is not the same as the function composition $f(g(x))$ because

____________________ $\neq$ ____________________.

9.3 One-to-One

Definition

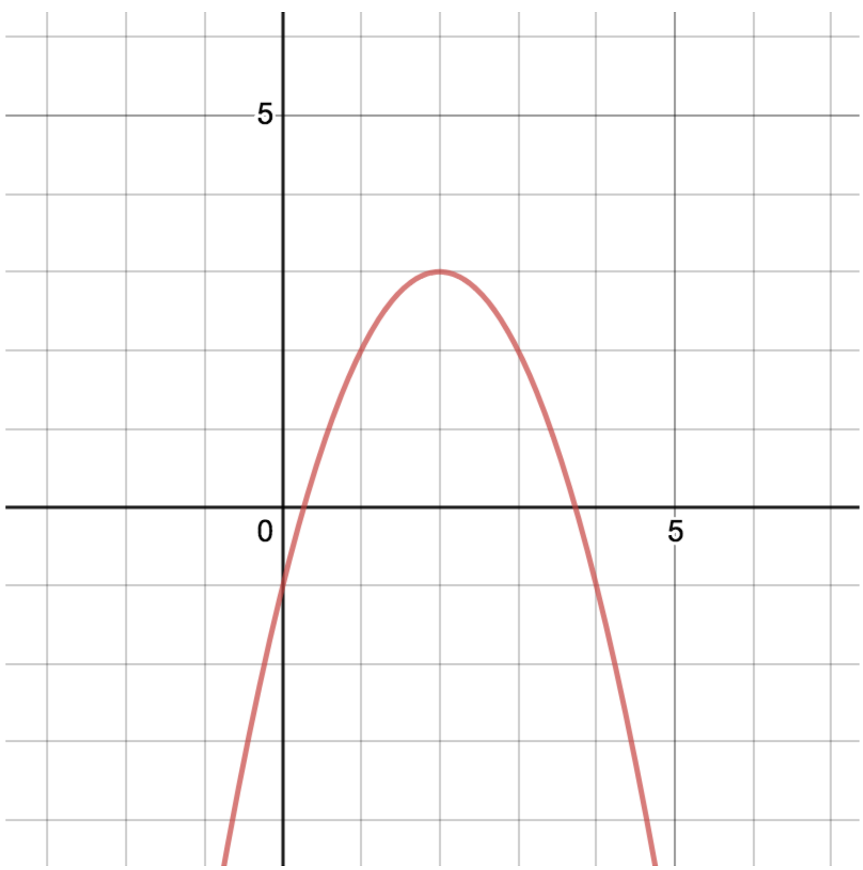
A function f is a ____________ - _____ - ____________ function if each value of the dependent variable (y) corresponds to exactly one value of the independent variable (x).

Note 4. If a function f is a set of ordered pairs, then f is one-to-one of no two ordered pairs have the same second element. That is, if each y has only one x.

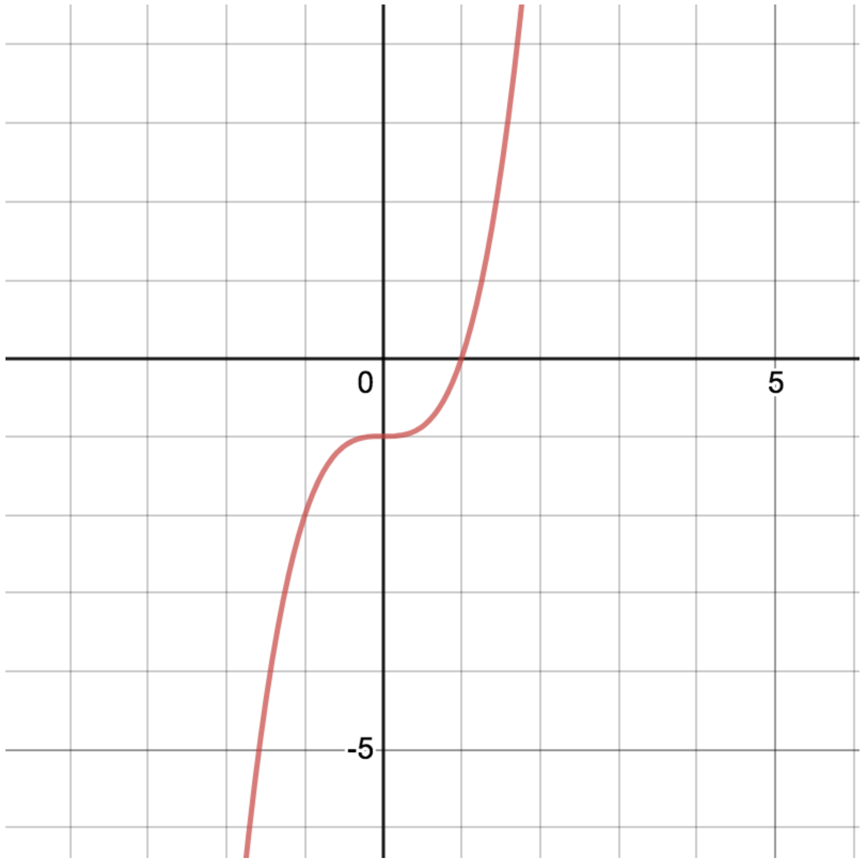
Horizontal Line Test

A function f is one-to-one if and only if any horizontal line intersects the graph of f at most once.

Example 7. Is the following graph one-to-one?



Example 8. Is the following graph one-to-one?



Algebraically Determine if a Function is One-to-One

To show that a function is one-to-one, you can show that $f(y) = f(x)$ if and only if

________________.

Example 9. Is the following function one-to-one?

$$f(x) = 3x + 4$$

Example 10. Is the following function one-to-one?

$$f(x) = x^3 - 1$$

Example 11. Is the following function one-to-one?

$$f(x) = \sqrt{x-1} + 3$$

9.4 Inverse

Definition

For any one-to-one function $f(x) = y$, a function $f^{-1}(x)$ is an ________________________ ________________________ of f if ________________. This can also be written as $f^{-1}(f(x)) = x$ for all x in the domain of f. It also follows that $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1}.

Note 5. Not every function has an inverse, and $f^{-1}(x) \neq$ ______. Given a one-to-one function, f, the inverse of the coordinate pair $(x, f(x))$ is ________________.

Example 12. For a particular one-to-one function $f(2) = 4$ and $f(5) = 12$, what are the corresponding input and output values for the inverse function?

How to Determine if Two Functions $f(x)$ and $g(x)$ are Inverses of Each Other

1. Determine whether ________________ or ________________.
2. If either statement is true, then both are true, and $g = f^{-1}$ and $f = g^{-1}$. If either statement is false, then both are false, and $g \neq f^{-1}$ and $f \neq g^{-1}$.

Domain and Range of Inverse Functions

The ________________ of a function $f(x)$ is the domain of the inverse function $f^{-1}(x)$. The ________________ of $f(x)$ is the range of $f^{-1}(x)$.

How to Find the Domain and Range of an Inverse Function

1. If the original function is one-to-one, write the range of the original function as the ________________ of the inverse function.

2. If the original function is one-to-one, write the domain of the original function as the ________________ of the inverse function.

3. If the domain of the original function needs to be restricted to make it one-to-one, then this restricted domain becomes the ________________ of the inverse function.

How to Determine the Inverse of a Function

1. Check that f is a ________________ - _____ - ________________ function.

2. Solve for x.

3. Interchange x and y.

Example 13. Determine if the following function is one-to-one. If the function is one-to-one, find the inverse and define the domain on which the inverse is valid:

$$f(x) = (4x - 5)^3 - 4$$

Example 14. Determine if the following function is one-to-one. If the function is one-to-one, find the inverse and define the domain on which the inverse is valid:

$$f(x) = \sqrt{-3x - 5} + 7$$

Example 15. Determine if the following function is one-to-one. If the function is one-to-one, find the inverse and define the domain on which the inverse is valid:

$$f(x) = (2x + 7)^2 + 2$$