Lecture 29: Section 4.6
Graphs of Other Trigonometric Functions

Secant function

Cosecant funtion

Tangent function

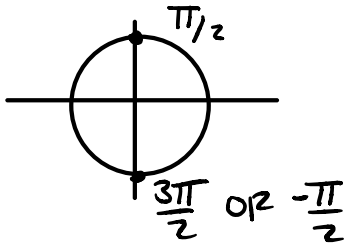
Cotangent function

"ALL REAL NUMBERS EXCEPT $\frac{\pi}{2} + n\pi$"

The functions secant and cosecant have period 2π: ★

$$\sec(x + 2\pi) = \sec x$$

$$\csc(x + 2\pi) = \csc x$$



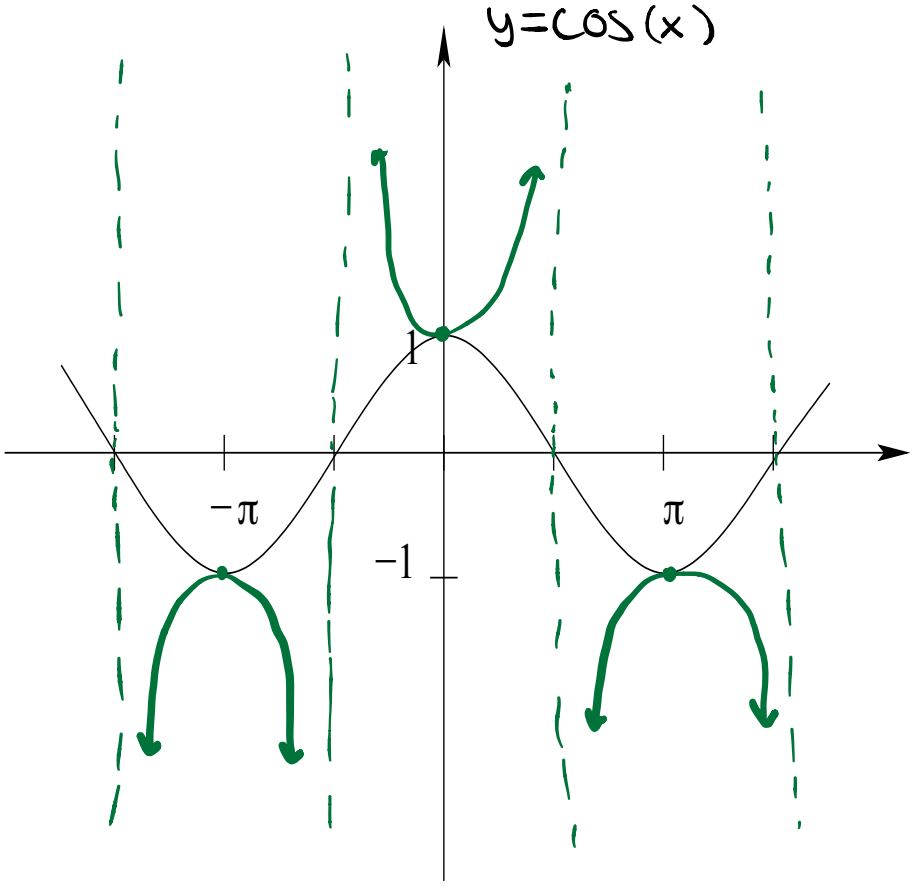
The Graph of $y = \sec x = \dfrac{1}{\cos x}$ CANT BE 0

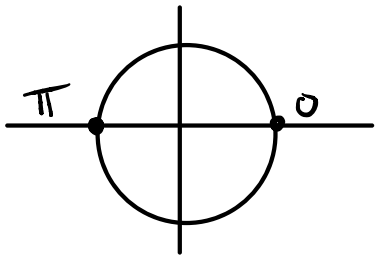
Domain: $x \neq \frac{\pi}{2} + n\pi$, n IS AN INTEGER

Range: $(-\infty, -1] \cup [1, \infty)$

Vertical Asymptotes: $x = \frac{\pi}{2} + n\pi$

Period: 2π





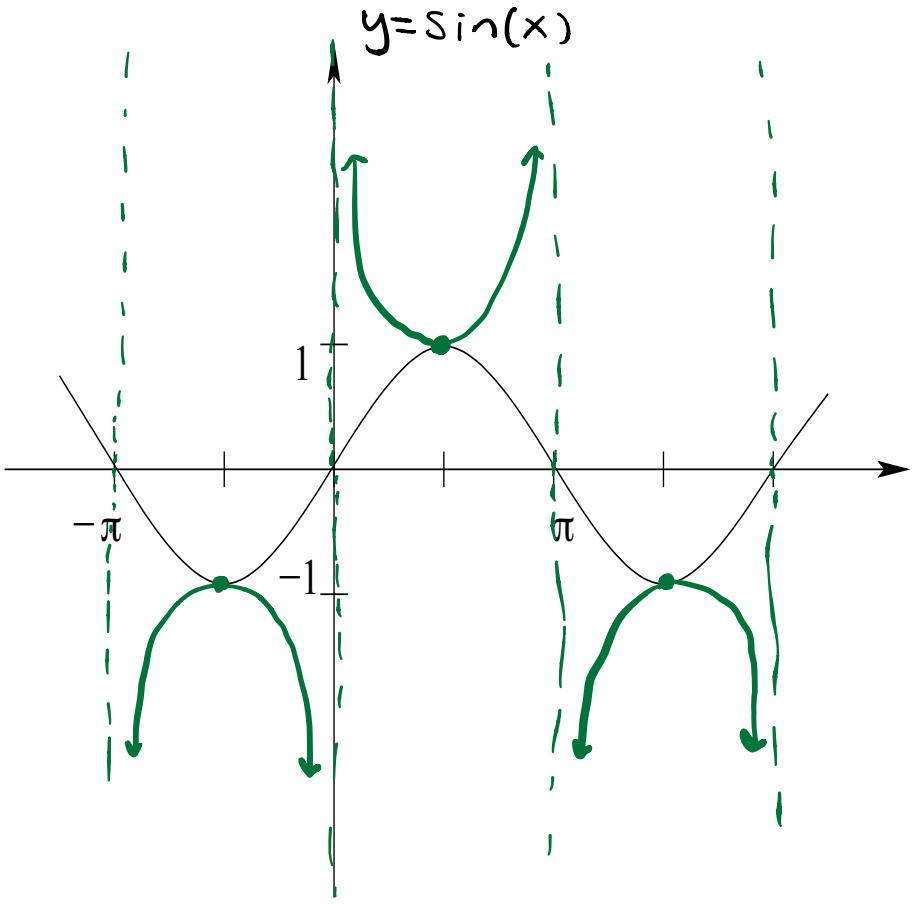
The Graph of $y = \csc x = \dfrac{1}{\sin x}$ CANT BE ZERO!

Domain: $x \neq n\pi$, n IS AN INTEGER

Range: $(-\infty, -1] \cup [1, \infty)$

Vertical Asymptotes: $x = n\pi$

Period: 2π



Checkpoint: Lecture 29, problem 1

ex. Find the period and vertical asymptotes for each function and sketch its graph. GRAPH $y = \frac{3}{2}\sin(2x+\pi)$

1) $y = \frac{3}{2}\csc(2x+\pi)$

NEW START: $2x+\pi=0 \Rightarrow x = \frac{-\pi}{2}$

NEW END: $2x+\pi=2\pi \Rightarrow x = \frac{\pi}{2}$

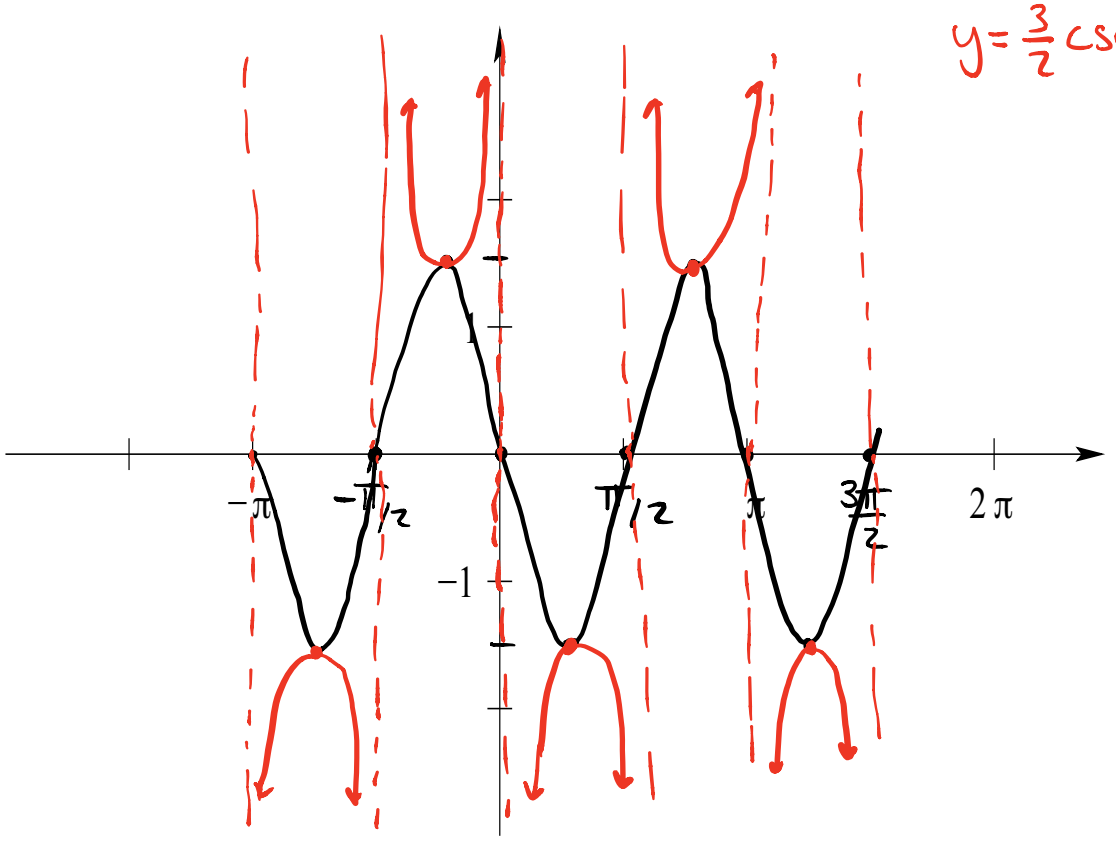
$\frac{1}{2}\left(-\frac{\pi}{2}+\frac{\pi}{2}\right)=0$

$\frac{1}{2}\left(-\frac{\pi}{2}+0\right)=\frac{1}{2}\left(-\frac{\pi}{2}\right)=\frac{-\pi}{4}$

$\frac{1}{2}\left(0+\frac{\pi}{2}\right)=\frac{1}{2}\left(\frac{\pi}{2}\right)=\frac{\pi}{4}$

X	$-\pi/2$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\pi/2$
Y	0	$\frac{3}{2}$	0	$-\frac{3}{2}$	0
	0	1	0	-1	0

$y=\frac{3}{2}\csc(2x+\pi)$



AMPLITUDE $= \frac{3}{2}$

$\boxed{\text{PERIOD} = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi}$

$\boxed{\text{VERTICAL ASYMPTOTES: } x = \frac{\pi}{2}n}$

GRAPH $y = \frac{1}{2}\cos\left(\frac{2x}{3} - \frac{\pi}{3}\right)$

2) $y = \frac{1}{2}\sec\left(\frac{2x}{3} - \frac{\pi}{3}\right)$

NEW START:

$\frac{2x}{3} - \frac{\pi}{3} = 0 \Rightarrow \frac{2x}{3} = \frac{\pi}{3} \Rightarrow 2x = \pi$

$x = \frac{\pi}{2}$

$\frac{1}{2}\left(\frac{7\pi}{2} + \frac{\pi}{2}\right) = \frac{1}{2}\left(\frac{8\pi}{2}\right) = 2\pi$

$\frac{1}{2}\left(2\pi + \frac{\pi}{2}\right) = \frac{1}{2}\left(\frac{5\pi}{2}\right) = \frac{5\pi}{4}$

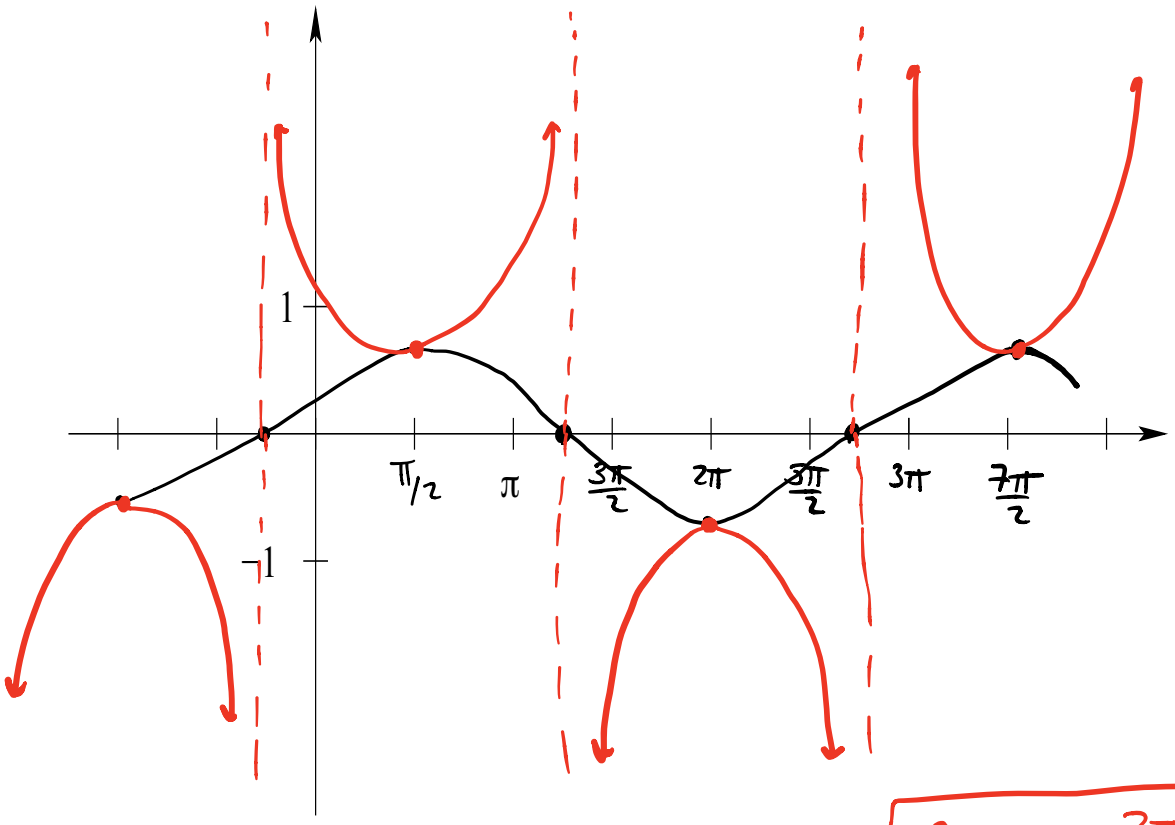
NEW END:

$\frac{2x}{3} - \frac{\pi}{3} = 2\pi \Rightarrow \frac{2x}{3} = \frac{7\pi}{3} \Rightarrow 2x = 7\pi$

$\Rightarrow x = \frac{7\pi}{2}$

$\frac{1}{2}\left(\frac{7\pi}{2} + 2\pi\right) = \frac{1}{2}\left(\frac{11\pi}{2}\right) = \frac{11\pi}{4}$

x	$\pi/2$	$\frac{5\pi}{4}$	2π	$\frac{11\pi}{4}$	$7\pi/2$
y	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
	1	0	-1	0	1



PERIOD $= \frac{2\pi}{b} = \frac{2\pi}{2/3}$

$= \frac{2\pi}{1} \cdot \frac{3}{2}$

$= 3\pi$

Checkpoint: Lecture 29, problem 2

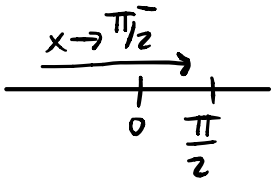
VERTICAL ASYMPTOTES: $x = \frac{5\pi}{4} + \frac{3\pi}{2}n$

$\frac{5\pi}{4}, \frac{11\pi}{4}$: $\frac{11\pi}{4} - \frac{6\pi}{4} = \frac{3\pi}{2}$

The functions tangent and cotangent have <u>period π</u>:

$$\tan(x+\pi) = \tan x$$

$$\cot(x+\pi) = \cot x$$

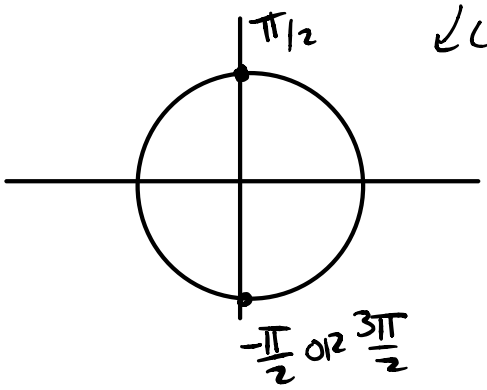


Vertical Asymptotes

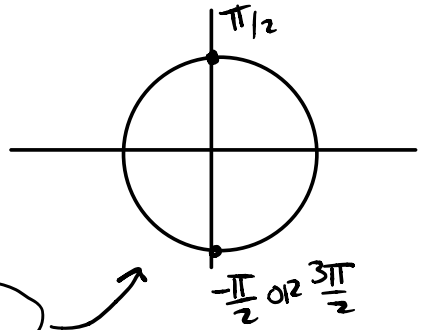
$$\tan x = \frac{\sin x}{\cos x} \longrightarrow \infty \qquad \text{as } x \to \frac{\pi}{2}^-$$

$$\tan x = \frac{\sin x}{\cos x} \longrightarrow \infty \qquad \text{as } x \to -\frac{\pi}{2}^+$$

Therefore, the vertical lines $x = \frac{\pi}{2}$ and $x = -\frac{\pi}{2}$ are vertical asymptotes of the graph of $y = \tan x$.



COS(x) IS ZERO HERE



The Graph of $y = \tan x = \dfrac{\sin(x)}{\cos(x)}$

CAN'T BE ZERO!

Domain: $x \neq \frac{\pi}{2} + n\pi$

Range: $(-\infty, \infty)$

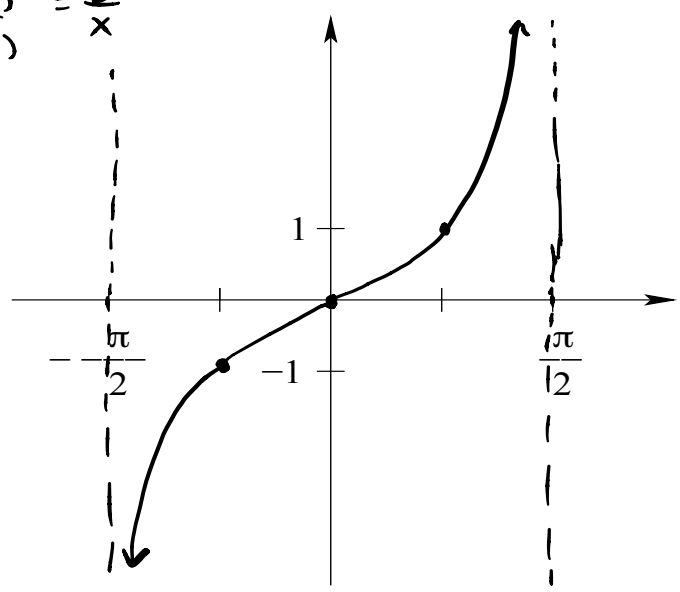
Vertical Asymptotes: $x = \frac{\pi}{2} + n\pi$

Period: π

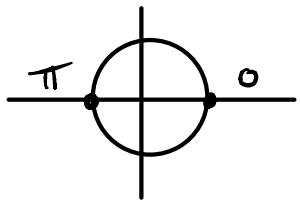
The key points in one period: $\tan(-\frac{\pi}{4}) = \dfrac{-\frac{\sqrt{2}}{2}}{\sqrt{2}/2} = -1$

x	$-\dfrac{\pi}{2}$	$-\dfrac{\pi}{4}$	0	$\dfrac{\pi}{4}$	$\dfrac{\pi}{2}$
$y = \tan x$	V.A.	-1	0	1	V.A.

$\tan(x) = \dfrac{\sin(x)}{\cos(x)} = \dfrac{y}{x}$



Checkpoint: Lecture 29, problem 3



The Graph of $y = \cot x = \frac{\cos(x)}{\sin(x)}$ CANT BE ZERO

Domain: $x \neq 0 + n\pi \Rightarrow x \neq n\pi$

Range: $(-\infty, \infty)$

Vertical Asymptotes: $x = n\pi$

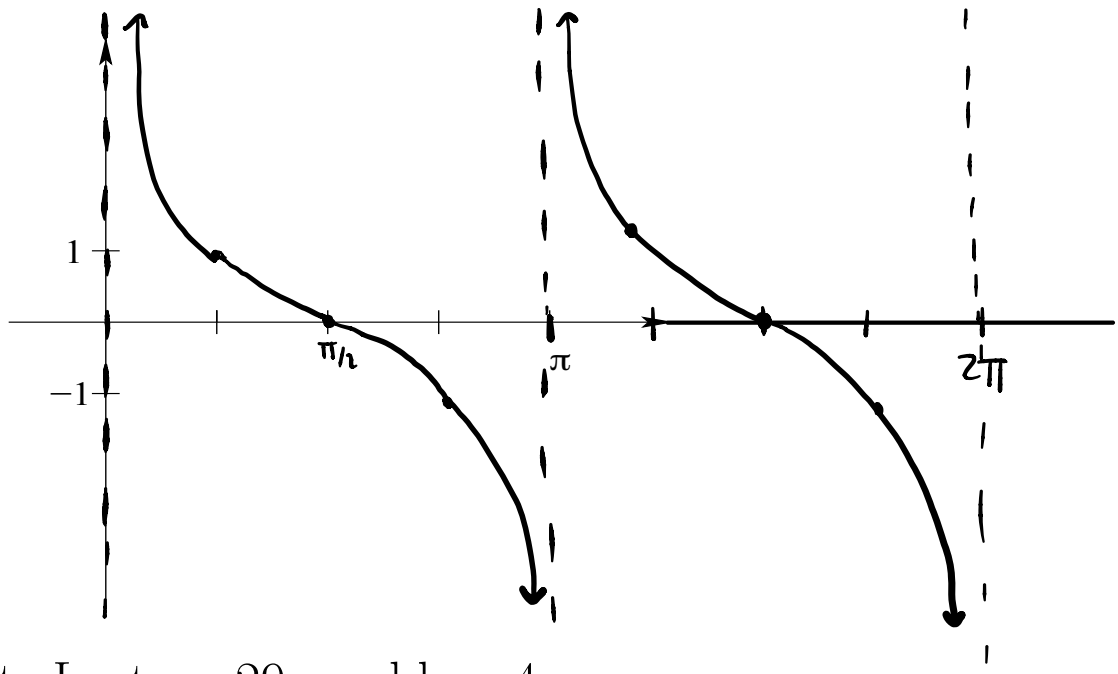
Period: π

The key points in one period:

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$y = \cot x$	V.A.	1	0	-1	V.A.

$\cot(\pi/4) = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

$\cot(\frac{3\pi}{4}) = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$



Checkpoint: Lecture 29, problem 4

Graphs of Transformations of the Tangent and Cotangent Functions

	$y = a\tan(bx - c)$	$y = a\cot(bx - c)$
Period	$P = \frac{\pi}{b}$	$P = \frac{\pi}{b}$
One cycle	$-\frac{\pi}{2} \le bx - c \le \frac{\pi}{2}$ $\frac{c}{b} - \frac{\pi}{2b} \le x \le \frac{c}{b} + \frac{\pi}{2b}$	$0 \le bx - c \le \pi$ $\frac{c}{b} \le x \le \frac{c}{b} + \frac{\pi}{b}$

ex. Find the period and vertical asymptotes for each function and sketch its graph. ① FIND V.A. BY SETTING ARGUMENT EQUAL TO $\frac{\pi}{2}$ AND $-\frac{\pi}{2}$:

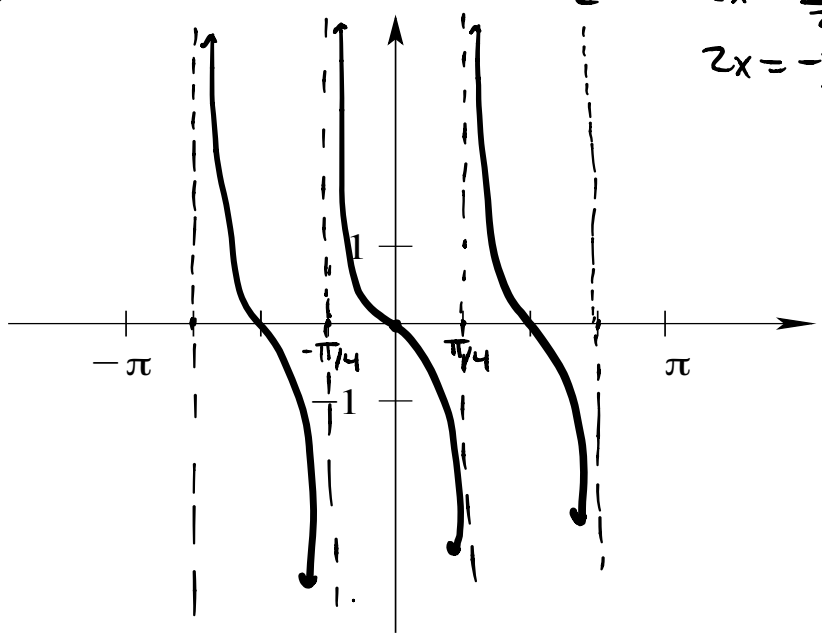
$2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$ (V.A.)

$2x = -\frac{\pi}{2} \Rightarrow x = -\frac{\pi}{4}$ (V.A.)

1) $y = -\tan(2x)$

REFLECTION OVER X-AXIS

PERIOD $= \frac{\pi}{b} = \frac{\pi}{2}$



VERTICAL ASYMPTOTES:

$x = \frac{\pi}{4} + \frac{\pi}{2}n$

PERIOD

(NOT GOING TO PAY TOO MUCH ATTENTION TO $\tan(2x)$)

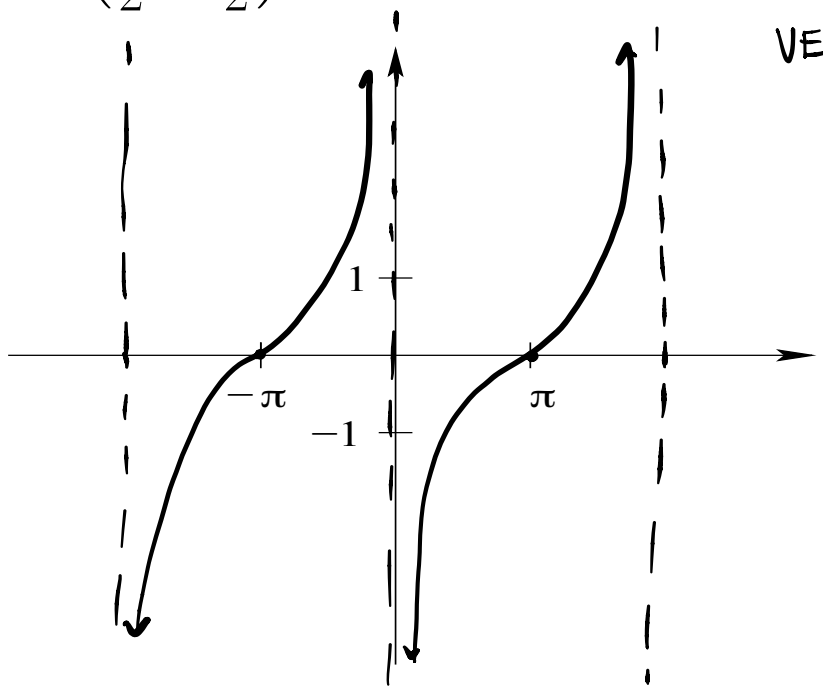
1. V.A.: SET ARGUMENT $= \frac{\pi}{2}, -\frac{\pi}{2}$

$\frac{x}{2} - \frac{\pi}{2} = \frac{\pi}{2}$ | $\frac{x}{2} - \frac{\pi}{2} = -\frac{\pi}{2}$

$\frac{x}{2} = \frac{2\pi}{2} \Rightarrow x = 2\pi$ | $\frac{x}{2} = 0 \Rightarrow x = 0$

2) $y = \tan\left(\frac{x}{2} - \frac{\pi}{2}\right)$

PERIOD $= \frac{\pi}{1/2} = 2\pi$



VERTICAL ASYMPTOTES:

$x = 0 + 2\pi n$

$x = 2\pi n$

0 + MULTIPLES OF PERIOD

3) $y = 2\cot\left(3x - \frac{\pi}{2}\right)$

VERTICAL STRETCH

FIND VERTICAL ASYMPTOTES BY SETTING ARGUMENT EQUAL TO 0, π

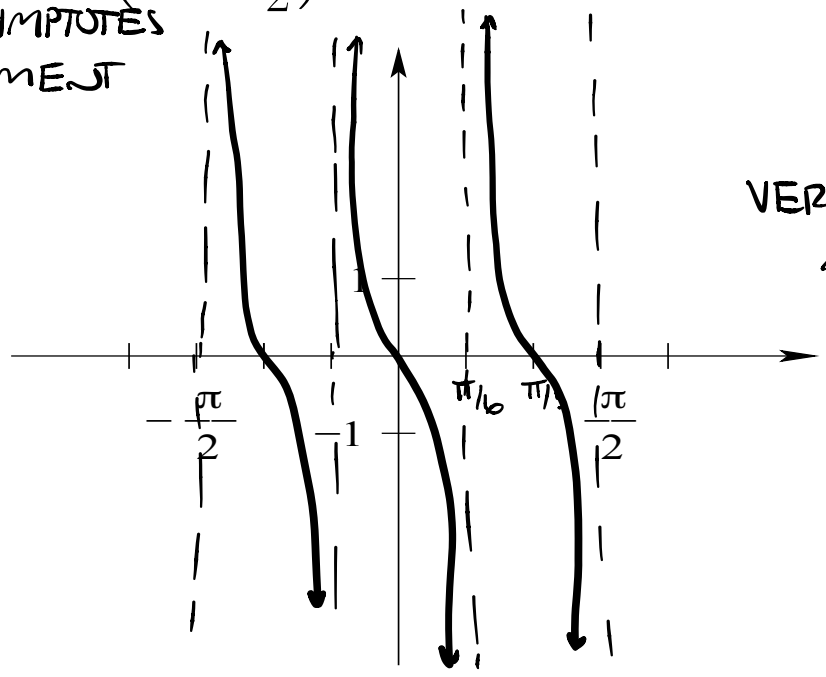
- $3x - \frac{\pi}{2} = 0$

  $3x = \frac{\pi}{2}$

  $x = \frac{\pi}{6}$

- $3x - \frac{\pi}{2} = \pi$

  $x = \frac{\pi}{2}$



PERIOD $= \frac{\pi}{b} = \frac{\pi}{3}$

VERTICAL ASYMPTOTES:

$x = \frac{\pi}{6} + \frac{\pi}{3} n$

Checkpoint: Lecture 29, problem 5