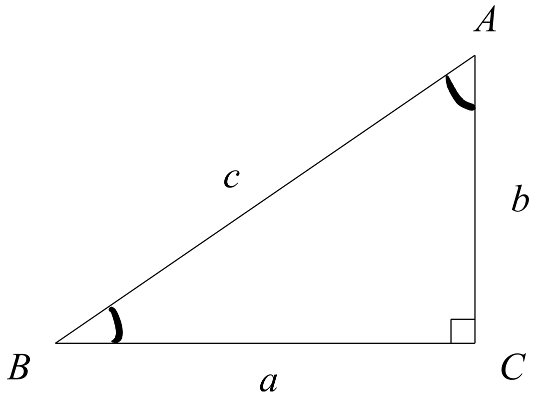
Lecture 31: Section 4.8
Applications and Models

Solving right triangles

Navigational bearings

Solving Right Triangles

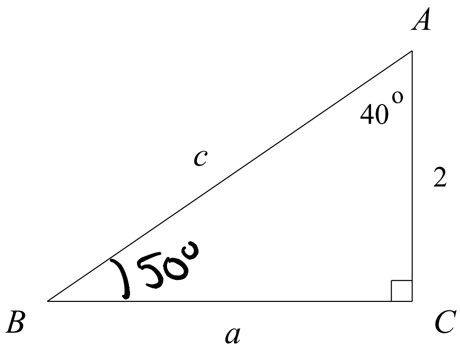
To solve a right triangle means to find the missing lengths of its sides and measurements of its angles.



$$a^2 + b^2 = c^2 \ ; \ A + B = 90^\circ$$

<u>**ex.**</u> Solve the triangle.

1)



$A + B = 90^\circ$

$40^\circ + B = 90^\circ$

$\boxed{B = 50^\circ}$

$\frac{OPP}{tan} = \tan(40^\circ)$

$\frac{a}{2} = \tan(40^\circ)$

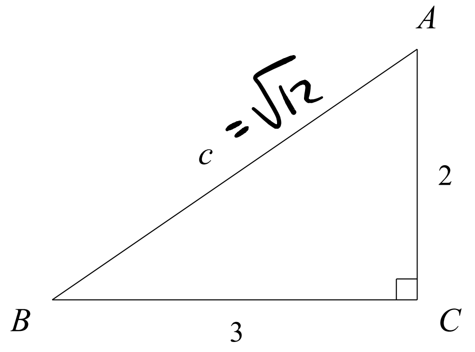
$\boxed{a = 2\tan(40^\circ) \approx 1.68}$

$\frac{ADJ}{HYP} = \cos(40^\circ)$

$\frac{2}{c} = \cos(40^\circ)$

$\boxed{c = \frac{2}{\cos(40^\circ)} = 2.61}$

2)

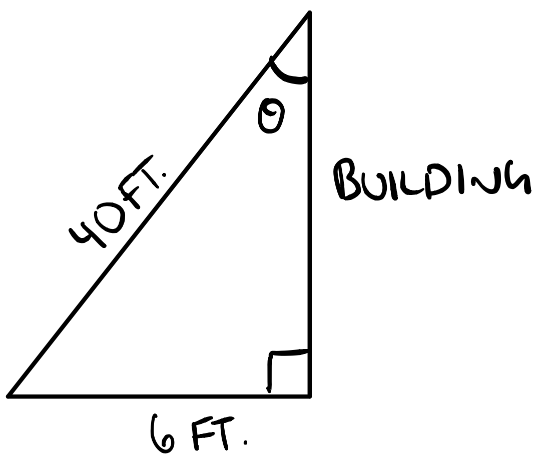


$$2^2 + 3^2 = c^2$$
$$4 + 9 = c^2$$
$$12 = c^2$$
$$\boxed{\sqrt{12} = c}$$

$$\tan(A) = \frac{3}{2} \Rightarrow \boxed{A = \tan^{-1}\left(\frac{3}{2}\right)}$$

$$\tan(B) = \frac{2}{3} \Rightarrow \boxed{B = \tan^{-1}\left(\frac{2}{3}\right)}$$

ex. A 40-ft ladder leans against a building. If the base of the ladder is 6 ft from the base of the building. What is the angle formed by the ladder and the building?



$$\frac{\text{OPP}}{\text{HYP}} = \sin(\theta)$$

$$\frac{6}{40} = \sin(\theta)$$

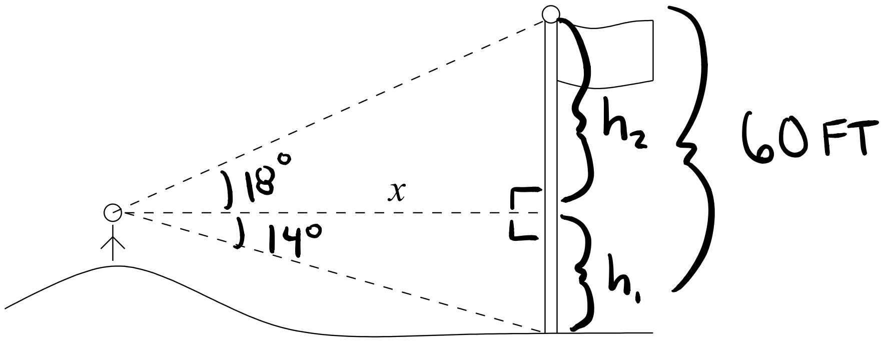
$$\frac{3}{20} = \sin(\theta)$$

$$\Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{3}{20}\right)} \approx 8.6^\circ$$

OR: $\boxed{\theta = \arcsin\left(\frac{3}{20}\right)}$

Checkpoint: Lecture 31, problem 1

ex. A woman standing on a hill sees a flagpole that she knows is 60 ft tall. The angle of depression to the bottom of the pole is 14°, and the angle of elevation to the top of the pole is 18°. Find her distance x to the pole.



$h_1 + h_2 = 60\text{FT}$

TOP Δ:

$$\frac{\text{OPP}}{\text{ADJ}} = \tan(18°)$$

$$\frac{h_2}{x} = \tan(18°)$$

$$\Rightarrow h_2 = x\tan(18°)$$

BOTTOM Δ:

$$\frac{\text{OPP}}{\text{ADJ}} = \tan(14°)$$

$$\frac{h_1}{x} = \tan(14°)$$

$$\Rightarrow h_1 = x\tan(14°)$$

$$h_1 + h_2 = 60$$

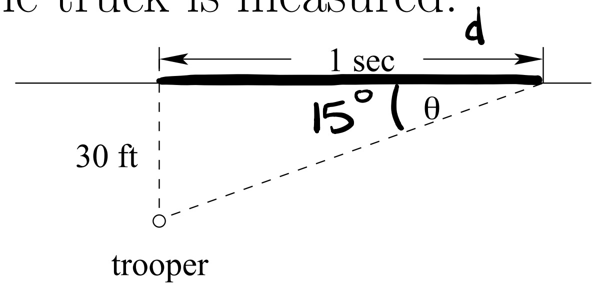
$$x\tan(14°) + x\tan(18°) = 60$$

$$x\left(\tan(14°) + \tan(18°)\right) = 60$$

$$\boxed{x = \frac{60}{\tan(14°)+\tan(18°)}}$$

$$x \approx 104\text{FT.}$$

ex. A state trooper is hidden 30 ft from a highway. One second after a truck passes, the angle between the highway and the line of observation from the patrol car to the truck is measured.



1) If the angle measures 15°, how fast is the truck traveling? Express the answer in feet per second and in miles per hour.

LET d = DISTANCE TRAVELED IN 1SEC

$$\frac{\text{OPP}}{\text{ADJ}} = \tan(15°)$$

$$\frac{30}{d} = \tan(15°) \Rightarrow d = \frac{30}{\tan(15°)} = \boxed{112 \text{ FT/SEC}}$$

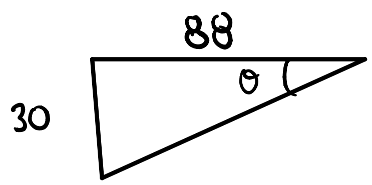
$$\frac{112\ \cancel{\text{FT}}}{1\ \cancel{\text{SEC}}} \cdot \frac{1\text{ MI}}{5280\ \cancel{\text{FT}}} \cdot \frac{3600\ \cancel{\text{SEC}}}{1\text{ HR}} = \frac{112 \cdot 3600\text{ MI}}{5280\text{ HR}} \approx \boxed{76\text{ MI/HR}}$$

2) If the speed limit is 55 miles per hour and a speeding ticket is issued for speeds of 5 miles per hour or more over the limit, for what angles should the trooper issue a ticket?

① NEED TO KNOW # OF FEET TRAVELED IN 1SEC GOING 60MPH

$$\frac{60\ \cancel{\text{MI}}}{1\ \cancel{\text{HR}}} \cdot \frac{5280\text{ FT}}{1\ \cancel{\text{MI}}} \cdot \frac{1\ \cancel{\text{HR}}}{3600\text{SEC}} = \frac{60 \cdot 5280}{3600}\text{ FT/SEC} \approx \boxed{88\text{ FT/SEC}}$$

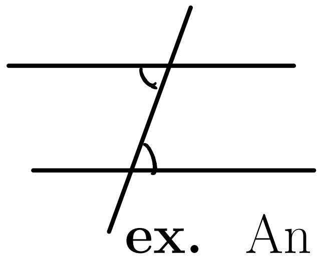
② FIND THE ANGLE ASSOCIATED WITH THE TRIANGLE



$$\frac{\text{OPP}}{\text{ADJ}} = \tan(\theta) \Rightarrow \tan(\theta) = \frac{30}{88}$$

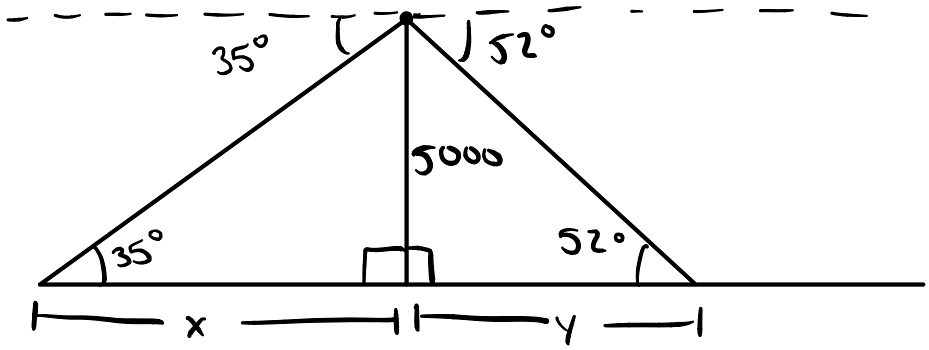
$$\Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{30}{88}\right) \approx 19°}$$

SO, IF $\theta < 19°$, YOU GET A TICKET



ALTERNATE INTERIOR ANGLES ARE CONGRUENT

ex. An airplane is flying at an elevation of 5000 ft, directly above a straight highway. Two motorists are driving cars on the highway on opposite sides of the plane, and the angle of depression to one car is 35° and to the other is 52°. How far apart are the cars?



FIND x+y

$$\tan(35°) = \frac{5000}{x} \Rightarrow \boxed{x = \frac{5000}{\tan(35°)}}$$

$$\tan(52°) = \frac{5000}{y} \Rightarrow \boxed{y = \frac{5000}{\tan(52°)}}$$

$$\Rightarrow \boxed{x+y = \frac{5000}{\tan(35°)} + \frac{5000}{\tan(52°)}}$$

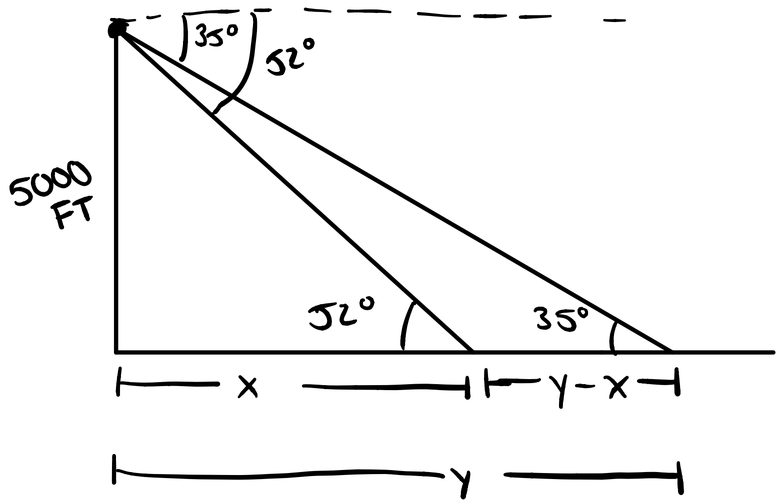
$$x+y \approx 3906\text{F} + 7141\text{ FT}$$

$$\approx 2 \text{ mi.}$$

Checkpoint: Lecture 31, problem 2

ex. If both cars in the previous example are on the side of the plane and if the angle of depression to one car is 35° and to the other is 52°, how far apart are the cars?

FIND $y-x$



$$\tan(52°) = \frac{5000}{x}$$

$$x = \frac{5000}{\tan(52°)}$$

$$\tan(35°) = \frac{5000}{y}$$

$$y = \frac{5000}{\tan(35°)}$$

$$\Rightarrow \boxed{y - x = \frac{5000}{\tan(35°)} - \frac{5000}{\tan(52°)}}$$

$\approx \frac{1}{2}$ MI

Checkpoint: Lecture 31, problem 3