Lecture 33: Section 5.2
Verifying Trigonometric Identities

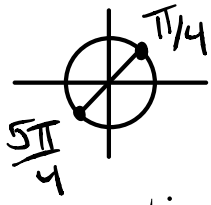
Verifying trigonometric identities

Conditional equation vs an identity

Ex: $x+3=5$

Recall: A **conditional equation** is an equation that is true for only some of the values in its domain.

ex. $\tan x = 1$



An **identity** is an equation that is true for all real values in its domain.

$x^2-9=(x+3)(x-3)$

NO MATTER WHAT X-VALUE YOU PLUG IN, THIS IS TRUE

ex. $\sin^2 x = 1 - \cos^2 x$

$\sin^2 x + \cos^2 x = 1$

Guidelines for Verifying Trigonometric Identities

1. Start with one side. It's usually easier to start with the more complicated side.

2. Use known identities. Bring fractional expressions to a common denominator, factor, and use the fundamental identities to simplify expressions.

3. Convert to sines and cosines. Sometimes it is helpful to rewrite all functions in terms of sines and cosines.

ex. Verify the trigonometric identities.

1) $\dfrac{1}{\sec x - \tan x} = \sec x + \tan x$

$*\tan^2 x + 1 = \sec^2 x$

$1 = \sec^2 x - \tan^2 x$

$$LHS = \left(\frac{1}{\sec x - \tan x}\right)\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right)$$

$$= \frac{\sec x + \tan x}{\sec^2 x - \tan^2 x}$$

$$= \frac{\sec x + \tan x}{1} = \sec x + \tan x = RHS \checkmark$$

2) $\dfrac{\sin^2\theta}{\cos\theta} = \sec\theta - \cos\theta$

$$RHS = \sec\theta - \cos\theta$$

$$= \frac{1}{\cos\theta} - \cos\theta = \frac{1}{\cos\theta} - \left(\frac{\cos\theta}{\cos\theta}\right)\cos\theta$$

$$= \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta} = LHS \checkmark$$

Checkpoint: Lecture 33, problem 1

ex. Verify the trigonometric identity

$$2\tan x \sec x = \frac{1}{1-\sin x} - \frac{1}{1+\sin x}$$

$$RHS = \frac{1}{1-\sin x} - \frac{1}{1+\sin x}$$

$$= \left(\frac{1+\sin x}{1+\sin x}\right)\left(\frac{1}{1-\sin x}\right) - \left(\frac{1-\sin x}{1-\sin x}\right)\left(\frac{1}{1+\sin x}\right)$$

$$= \frac{1+\sin x}{(1+\sin x)(1-\sin x)} - \frac{1-\sin x}{(1+\sin x)(1-\sin x)}$$

$$= \frac{(1+\sin x)-(1-\sin x)}{1-\sin^2 x}$$

$$= \frac{2\sin x}{\cos^2 x} = \frac{2\sin x}{\cos x} \cdot \frac{1}{\cos x} = 2\tan x \sec x = LHS \checkmark$$

ex. Verify the trigonometric identity

$$\frac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$$

$$\text{LHS} = \frac{\cos\theta}{1-\sin\theta} = \frac{\cos\theta}{1-\sin\theta}\left(\frac{1+\sin\theta}{1+\sin\theta}\right)$$

$$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta + \cos\theta\sin\theta}{\cos^2\theta}$$

$$= \frac{\cancel{\cos\theta}}{\cancel{\cos^2\theta}_{\cos\theta}} + \frac{\cancel{\cos\theta}\sin\theta}{\cancel{\cos^2\theta}_{\cos\theta}}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta$$

$$= \text{RHS} \checkmark$$

Checkpoint: Lecture 33, problem 2

ex. Verify the trigonometric identity

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\csc\theta$$

$$\text{LHS} = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{\sin\theta}{1+\cos\theta}\left(\frac{\sin\theta}{\sin\theta}\right) + \frac{1+\cos\theta}{\sin\theta}\left(\frac{1+\cos\theta}{1+\cos\theta}\right)$$

$$= \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)} + \frac{1+2\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta+1+2\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)} \qquad \sin^2\theta+\cos^2\theta=1$$

$$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2\cancel{(1+\cos\theta)}}{\sin\theta\cancel{(1+\cos\theta)}} = 2\left(\frac{1}{\sin\theta}\right)$$

$$= 2\csc\theta$$

$$= \text{RHS} \checkmark$$

ex. Verify the trigonometric identity

$$\frac{\tan\theta + \cot\theta}{\sec\theta\csc\theta} = 1$$

$$\text{LHS} = \frac{\tan\theta + \cot\theta}{\sec\theta\csc\theta} = \frac{\left(\frac{\sin\theta}{\cos\theta}\right) + \left(\frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)}$$

COMMON DENOM.

$$= \frac{\left(\frac{\sin\theta}{\sin\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right) + \left(\frac{\cos\theta}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)}$$

$$= \frac{\frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\cos\theta\sin\theta}}{\left(\frac{1}{\cos\theta\sin\theta}\right)} = \frac{\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}}{\frac{1}{\cos\theta\sin\theta}}$$

$$= \frac{1}{\cos\theta\sin\theta} \cdot \frac{\cos\theta\sin\theta}{1}$$

$$= 1 = \text{LHS} \checkmark$$

Checkpoint: Lecture 33, problem 3

Sometimes, it is practical to work with each side separately to obtain one common form equivalent to both sides.

ex. Verify the trigonometric identity

$$\frac{1+\cos\theta}{\cos\theta} = \frac{\tan^2\theta}{\sec\theta - 1}$$

$$\text{LHS} = \frac{1+\cos\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\cos\theta}{\cos\theta} = \sec\theta + 1$$

$$\text{RHS} = \frac{\tan^2\theta}{\sec\theta - 1}\left(\frac{\sec\theta+1}{\sec\theta+1}\right) = \frac{\tan^2\theta\,(\sec\theta+1)}{\sec^2\theta - 1}$$

$$= \frac{\cancel{\tan^2\theta}\,(\sec\theta+1)}{\cancel{\tan^2\theta}}$$

$$= \sec\theta + 1$$

$$= \text{LHS} \checkmark$$

Calculus Examples

ex. Verify:

1) $\sin^3 x \cos^2 x = (\cos^2 x - \cos^4 x)\sin x$

$$\text{LHS} = \sin^3 x \cos^2 x = \sin x(\sin^2 x \cos^2 x) \qquad \sin^2 x = 1 - \cos^2 x$$

$$= \sin x((1-\cos^2 x)\cos^2 x)$$

$$= \sin x(\cos^2 x - \cos^4 x)$$

$$= \text{RHS} \checkmark$$

2) $\sec^6 x \tan x = (\tan x + 2\tan^3 x + \tan^5 x)\sec^2 x$

$$\text{LHS} = \sec^6 x \tan x$$

$$= \sec^2 x(\sec^4 x \tan x)$$

$$= \sec^2 x((\sec^2 x)^2 \tan x) \qquad 1 + \tan^2 x = \sec^2 x$$

$$= \sec^2 x((1+\tan^2 x)^2 \tan x)$$

$$= \sec^2 x((1 + 2\tan^2 x + \tan^4 x)\tan x)$$

$$= \sec^2 x(\tan x + 2\tan^3 x + \tan^5 x) = \text{RHS} \checkmark$$

Checkpoint: Lecture 33, problem 4