Name: Solutions 9/6/22 MAC 2313 Quiz 1 (L1-5)

1. [2 points] Find the vector in the direction of $v = \langle 3, -1, 1 \rangle$ which has magnitude exactly 2.

$|v| = \sqrt{3^2 + (-1)^2 + 1^2} = \sqrt{11}$, so

$\hat{v} = \frac{v}{|v|} = \langle \frac{3}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{1}{\sqrt{11}} \rangle$ has length 1,

$2\hat{v} = \boxed{\langle \frac{6}{\sqrt{11}}, \frac{-2}{\sqrt{11}}, \frac{2}{\sqrt{11}} \rangle}$ has length 2

2. [2 points] Find a ***unit*** vector orthogonal to both $u = \langle 1, 2, 0 \rangle$ and $v = \langle 1, 0, 3 \rangle$.

$n = \langle 1, 2, 0 \rangle \times \langle 1, 0, 3 \rangle = \langle 6, -3, -2 \rangle$ is orthogonal to both, so is any scalar multiple

$|n| = \sqrt{36 + 9 + 4} = \sqrt{49} = 7$,

$\hat{n} = \frac{n}{|n|} = \boxed{\langle \frac{6}{7}, -\frac{3}{7}, -\frac{2}{7} \rangle}$

2. [4 Points] Find an equation of the plane which is parallel to both vectors $u = \langle 1, 2, 3\rangle$ and $v = \langle -1, 0, -1\rangle$, and which contains the point $(0, 0, 5)$.

$n = u \times v$ is orthogonal to both

$= \langle 1, 2, 3\rangle \times \langle -1, 0, -1\rangle = \begin{vmatrix} 2 & 3 \\ 0 & -1 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 3 \\ -1 & -1 \end{vmatrix} j + \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix} k = \langle -2, -2, 2\rangle,$

$-2x - 2y + 2z + d = 0$ is equation.

$(0, 0, 5)$ satisfies, so

$0 + 0 + 2(5) + d = 0$

$\Rightarrow d = -10$

$\boxed{-2x - 2y + 2z - 10 = 0}$

or $\boxed{x + y - z + 5 = 0}$