Name: Sol'ns 9/13/22 MAC 2313 Quiz 2 (L6-7)

1. Consider the surface given by the equation

$$x^2 - y^2 + z^2 - 2x + 6z + 9 = 0$$

(a) [1 point] Complete the square to write the equation in one of the standard forms.

(b) [1 point] Classify and find an equation for the traces parallel to the xy-plane.

(c) [1 point] Classify and find an equation for the traces parallel to the yz-plane.

(d) [1 point] Classify and find an equation for the traces parallel to the xz-plane.

a) $(x^2 - 2x + 1) - y^2 + (z^2 + 6z + 9) = 0 + 1$

$$(x-1)^2 - y^2 + (z+3)^2 = 1$$

b) Parallel to xy-plane $\Rightarrow$ $z = K$ for some real number K

$(x-1)^2 - y^2 = 1 - (K+3)^2$ is a hyperbola

c) $x = K$ for some K, $-y^2 + (z+3)^2 = -(K-1)^2 + 1$ is also a hyperbola

d) $y = K$, $(x-1)^2 + (z+3)^2 = 1 + K^2$ is an ellipse, or more specifically, a circle

CHECK BACK

2. Consider the parametric space curve given by

$$\mathbf{r}(t) = \langle 2\ln(t),\, t^3,\, t^2 + 4t - 5\rangle$$

(a) [2 points]. Find the first derivative $\mathbf{r}'(t)$
(b) [2 points]. At $t = 1$, find the unit tangent vector $\hat{\mathbf{T}}(1)$

a) $r'(t) = \boxed{\left\langle 2\cdot\frac{1}{t},\ 3t^2,\ 2t+4 \right\rangle}$

b) $\hat{T}(t) = \dfrac{r'(t)}{|r'(t)|}$, $\hat{T}(1) = \dfrac{r'(1)}{|r'(1)|} = \dfrac{\langle 2, 3, 6\rangle}{\sqrt{2^2 + 3^2 + 6^2}}$

$= \dfrac{\langle 2, 3, 6\rangle}{\sqrt{4+9+36}} = \dfrac{\langle 2, 3, 6\rangle}{\sqrt{49}} = \boxed{\left\langle \frac{2}{7}, \frac{3}{7}, \frac{6}{7} \right\rangle}$