Name: Solutions 10/4/22 MAC 2313 Quiz 4 (L11-13)

1. [4 points] For each of the following, either evaluate the limit, or show it does not exist. These have many ways of showing limit DNE

(a) $\lim_{(x,y)\to(0,0)} \frac{3x+2y}{2x-3y}$

Take $x=0$, then $y \to 0$,

$$\lim_{y\to 0} \frac{2y}{-3y} = -\frac{2}{3}$$

Take $y=x$, then as $x \to 0$,

$$\lim_{x\to 0} \frac{3x+2x}{2x-3x} = \lim_{x\to 0} \frac{5x}{-x} = -5$$

$-5 \neq -\frac{2}{3}$, so limit $\boxed{DNE}$

(b) $\lim_{(x,y)\to(0,0)} \frac{3x\sqrt{x^2+y^2}}{x^2+y^2}$ Convert to Polar,

$$= \lim_{r\to 0} \frac{3r\cos\theta \cdot \sqrt{r^2}}{r^2} = \lim_{r\to 0} \frac{3r^2\cos\theta}{r^2} = 3\cos\theta.$$

the limit depends on the angle of approach, $\theta$, so the limit $\boxed{DNE}$

CHECK BACK

2. [4 points] Let $f(x, y) = 5x\cos(y) + (x+2)^2\sin(y)$

(a) Find the first partials $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$

$$\frac{\partial f}{\partial x} = 5\cos(y) + 2(x+2)\sin(y)$$

$$\frac{\partial f}{\partial y} = -5x\sin(y) + (x+2)^2\cos(y)$$

(b) Find the equation of the tangent plane to $f$ at the point $(1, 0, 5)$

$$f_x(1,0) = 5\cdot\cos(0) + 2(1+2)\sin(0) = 5$$

$$f_y(1,0) = -5\cdot 1\cdot\sin(0) + (1+2)^2\cos(0) = 3^2 = 9.$$

tangent plane: $z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$$\boxed{z - 5 = 5(x-1) + 9(y-0)}$$ ← This is fine,

or $\boxed{z = 5 + 5x - 5 + 9y}$ or,

or $\boxed{z = 5x + 9y}$ or