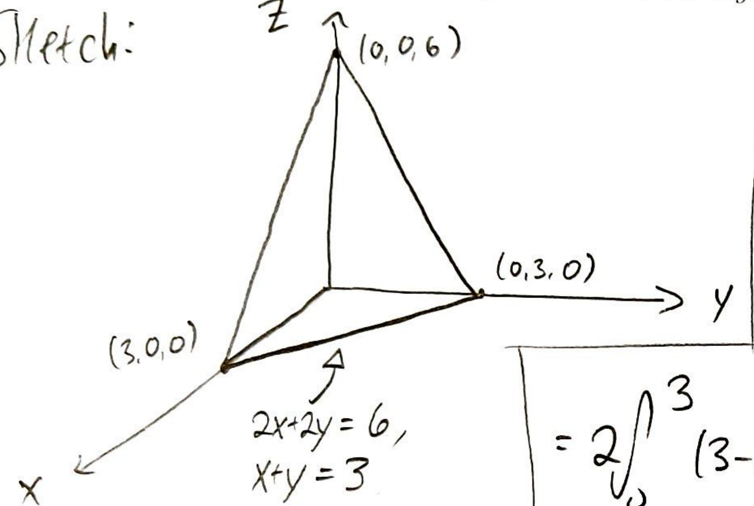
Name: Solutions 10/25/22 MAC 2313 Quiz 6 (L19-20)

1. [4 Points] Find the volume of the region bounded underneath the plane with equation

$$z = 6 - 2x - 2y$$

within the first octant. [*Hint: sketch the region in 3D*]

Sketch:



$2x+2y=6$,
$x+y=3$
$y=3-x$

$$\boxed{V=\int_0^3\int_0^{3-x} 6-2x-2y \, dy\, dx}$$

$$=2\int_0^3\int_0^{3-x} (3-x)-y \, dy\, dx$$

$$=2\int_0^3 (3-x)y-\frac{y^2}{2}\Big|_{y=0}^{y=3-x} dx$$

$$=2\int_0^3 (3-x)(3-x)-\frac{(3-x)^2}{2}\,dx = 2\int_0^3 \frac{(3-x)^2}{2}\,dx = \int_0^3 (3-x)^2\,dx$$

$$= -\frac{(3-x)^3}{3}\Big|_0^3 = \frac{3^3}{3} = \boxed{9}$$

Alternatively: Volume of pyramid $V=\frac{1}{3}Bh$, $B=$ area of base, $h=$ height.

B is triangle, $B=\frac{1}{2}lw=\frac{1}{2}(3)(3)=\frac{9}{2}$

h is height, $h=6$. So Volume is $V=\frac{1}{3}\left(\frac{9}{2}\right)\cdot 6=\boxed{9}$

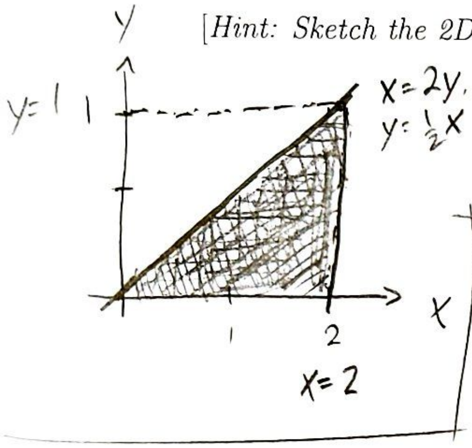
CHECK BACK

2. [4 points] Evaluate the following integral:

Sketch:

$$\int_0^1 \int_{2y}^2 3e^{x^2}\, dx\, dy$$

[*Hint: Sketch the 2D region and reverse the order of integration*]



$0 \le y \le \frac{1}{2}x$
$0 \le x \le 2$, So integral $= \int_0^2 \int_0^{\frac{1}{2}x} 3e^{x^2}\, dy\, dx$

$= \int_0^2 3e^{x^2}\left(\frac{1}{2}x - 0\right) dx$

$= \frac{3}{2}\int_0^2 x e^{x^2}\, dx$

$u = x^2$
$du = 2x\,dx$,
$\frac{1}{2}du = x\,dx$

$x=0 \Rightarrow u=0$
$x=2 \Rightarrow u=4$

$= \frac{3}{2}\int_{u=0}^{u=4} \frac{1}{2} e^u\, du$

$= \frac{3}{4}\int_0^4 e^u\, du = \frac{3}{4}e^u \Big|_0^4 = \boxed{\frac{3}{4}\left(e^4 - 1\right)}$

Swapping the order is necessary: e^{x^2} has no elementary antiderivative (with respect to x)