SETS AND LOGIC (MHF 3202) – PRACTICE MIDTERM 1

Equivalences

(1) (De Morgan's laws)
$\neg(P \wedge Q)$ is equivalent to $\neg P \vee \neg Q$.
$\neg(P \vee Q)$ is equivalent to $\neg P \wedge \neg Q$.

(2) (Commutative laws)
$P \wedge Q$ is equivalent to $Q \wedge P$.
$P \vee Q$ is equivalent to $Q \vee P$.

(3) (Associative laws)
$P \wedge (Q \wedge R)$ is equivalent to $(P \wedge Q) \wedge R$.
$P \vee (Q \vee R)$ is equivalent to $(P \vee Q) \vee R$.

(4) (Idempotent laws)
$P \wedge P$ is equivalent to P.
$P \vee P$ is equivalent to P.

(5) (Distributive laws)
$P \wedge (Q \vee R)$ is equivalent to $(P \wedge Q) \vee (P \wedge R)$.
$P \vee (Q \wedge R)$ is equivalent to $(P \vee Q) \wedge (P \vee R)$.

(6) (Absorption laws)
$P \vee (P \wedge Q)$ is equivalent to P.
$P \wedge (P \vee Q)$ is equivalent to P.

(7) (Double Negation law)
$\neg\neg P$ is equivalent to P.

(8) (Tautology laws)
$P\wedge$ (a tautology) is equivalent to P.
$P\vee$ (a tautology) is a tautology.
$\neg$(a tautology) is a contradiction.

(9) (Contradiction laws)
$P\wedge$ (a contradiction) is a contradiction.
$P\vee$ (a contradiction) is euqivalent to P.
$\neq$(a contradiction) is a tautology.

(10) (Conditional law)
$P \to Q$ is equivalent to $\neg P \vee Q$.

Question 1. *Circle the correct answer.*

(1) *(1 point)*

Is the following valid reasoning?

"If I answer this question correctly, I will get a point. Therefore I will get a point."

True.

False.

(2) *(1 point)*

Is the formula $(P \vee Q) \wedge \neg Q$ *equivalent to* $P \wedge \neg Q$*?*

True.

False.

(3) *(1 point)*

Is the emptyset the truth set for the statement $P(x)$ *saying "*x *is an even prime number." in the universe of natural numbers?*

True.

False.

(4) *(1 point)*

Let A, B, C *be sets. Is the following equality true?*

$$A \setminus B \ \cup \ B \setminus C = A \setminus C$$

True.

False.

(5) *(1 point)*

Is there more than one free variable in the following statement?

$$w \in \{x : x = 2k + 1\}$$

True.

False.

Question 2. *(10 points) Use a truth table to verify the Absorption law* $P \vee (P \wedge Q)$ *is equivalent to* P*.*

Question 3. *(15 point) Let A, B, C be sets. Use logical equivalences to show that*

$$(A \setminus B) \cap C = (A \cap C) \setminus (B \cap C).$$

Question 4. *(15 points) Make a truth table for the formula*

$$((P \to Q) \wedge (Q \to R)) \to (P \to R).$$

Decide whether the formula above is a tautology, contradiction, or neither.

Question 5.

(a) *(2 points) Analyze the logical form of the following statement: Both having a fever and having a headache are sufficient conditions for George to go to the doctor.*

(b) *(3 points) Translate the following statement into idiomatic English.*

$$\exists x \forall y P(x, y),$$

where $P(x, y)$ means "x is a parent of y".