# SETS AND LOGIC (MHF 3202) – PRACTICE MIDTERM 2

## Equivalences

(1) (De Morgan's laws)
 $\neg(P \wedge Q)$ is equivalent to $\neg P \vee \neg Q$.
 $\neg(P \vee Q)$ is equivalent to $\neg P \wedge \neg Q$.

(2) (Commutative laws)
 $P \wedge Q$ is equivalent to $Q \wedge P$.
 $P \vee Q$ is equivalent to $Q \vee P$.

(3) (Associative laws)
 $P \wedge (Q \wedge R)$ is equivalent to $(P \wedge Q) \wedge R$.
 $P \vee (Q \vee R)$ is equivalent to $(P \vee Q) \vee R$.

(4) (Idempotent laws)
 $P \wedge P$ is equivalent to $P$.
 $P \vee P$ is equivalent to $P$.

(5) (Distributive laws)
 $P \wedge (Q \vee R)$ is equivalent to $(P \wedge Q) \vee (P \wedge R)$.
 $P \vee (Q \wedge R)$ is equivalent to $(P \vee Q) \wedge (P \vee R)$.

(6) (Absorption laws)
 $P \vee (P \wedge Q)$ is equivalent to $P$.
 $P \wedge (P \vee Q)$ is equivalent to $P$.

(7) (Double Negation law)
 $\neg\neg P$ is equivalent to $P$.

(8) (Tautology laws)
 $P\wedge$ (a tautology) is equivalent to $P$.
 $P\vee$ (a tautology) is a tautology.
 $\neg$(a tautology) is a contradiction.

(9) (Contradiction laws)
 $P\wedge$ (a contradiction) is a contradiction.
 $P\vee$ (a contradiction) is euqivalent to $P$.
 $\neq$(a contradiction) is a tautology.

(10) (Quantifier Negation laws)
 $\neg\exists x P(x)$ is equivalent to $\forall x \neg P(x)$.
 $\neg\forall x P(x)$ is equivalent to $\exists x \neg P(x)$.

**Question 1.** *Circle the correct answer.*

(1) *(1 point)*

*Let $I$ be an index set. Suppose that for every $i \in I$ we have $A_i \subseteq B_i$. Then $\bigcup_{i\in I} A_i \subseteq \bigcup_{i\in I} B_i$.*

*True.*

*False.*

*Let $n$ be an integer. Then $n^2 - n + 1$ is even.*

(2) *True.*

*False.*

(3) *(1 point)*

*There are exactly two real numbers $x$ such that $x^2 = x$.*

*True.*

*False.*

*If $A \cap B = \emptyset$ then $A \not\subseteq B$ and $B \not\subseteq A$.*

(4) *True.*

*False.*

(5) *For any real number $x$, $x^3 > 0$ if and only if $x > 0$.*

*True.*

*False.*

**Question 2.** *For integers $m$ and $n$, let $m|n$ denote that $m$ divides $n$. Prove that $\{n \in \mathbb{Z} : 12|n\} \subseteq \{n \in \mathbb{Z} : 3|n \vee 4|n\}$.*

**Question 3.** *Suppose that $\mathcal{F}$ and $\mathcal{G}$ are non-empty families of sets. Prove that if $\mathcal{F} \subset \mathcal{G}$ then $\bigcap G \subset \bigcap F$.*

**Question 4.** *Suppose $x$ and $y$ are real numbers and $x \neq 0$. Prove that $y + \frac{1}{x} = 1 + \frac{y}{x}$ if and only if $x = 1$ or $y = 1$.*

**Question 5.** *Prove that there is a unique set $A$ such that $A \Delta B = B = B \Delta A$ for every set $B$.*