Homework 19

4) $\lim_{x\to 0} \frac{\tan(x)-x}{x^3} = \lim_{x\to 0} \frac{\sec^2 x - 1}{3x^2} = \lim_{x\to 0} \frac{2\sec x \cdot \sec x \cdot \tan x}{6x}$

$= \lim_{x\to 0} \frac{\sec^2 x \tan x}{3x} = \lim_{x\to 0} \frac{\sec^2 x \cdot \sec^2 x + \tan x \cdot 2\sec^2 x \tan x}{3}$

$= \lim_{x\to 0} \frac{\sec^4 x + 2\sec^2 x \tan x}{3} = \frac{1+0}{3} = \frac{1}{3}$

8) $\lim_{x\to -\infty} x \ln\left(1-\frac{1}{x}\right) = \lim_{x\to -\infty} \frac{\ln\left(1-\frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x\to -\infty} \frac{\frac{1}{1-\frac{1}{x}}\left(\frac{1}{x^2}\right)}{-\frac{1}{x^2}}$

$= \lim_{x\to -\infty} -\frac{1}{1-\frac{1}{x}} = -1$

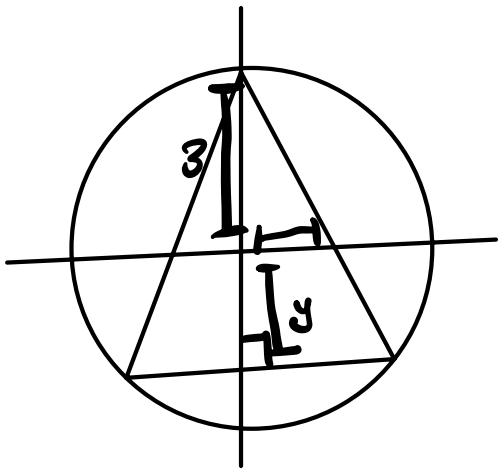
13) $\lim_{x\to 1} (2-x)^{\tan\left(\frac{\pi}{2}x\right)} = \lim_{x\to 1} e^{\ln\left[(2-x)^{\tan\left(\frac{\pi}{2}x\right)}\right]}$

$= \lim_{x\to 1} e^{\tan\left(\frac{\pi}{2}x\right)\ln(2-x)} = \lim_{x\to 1} e^{\frac{\ln(2-x)}{\cot\left(\frac{\pi}{2}x\right)}}$

$= \lim_{x\to 1} e^{\frac{-\frac{1}{2-x}}{-\csc^2\left(\frac{\pi}{2}x\right)\cdot\frac{\pi}{2}}} = e^{\frac{\frac{1}{1}}{\csc^2\left(\frac{\pi}{2}\right)\cdot\frac{\pi}{2}}} = e^{\frac{1}{\frac{\pi}{2}}} = e^{\frac{2}{\pi}}$

Homework 21

6) max area isosceles Δ in $\odot$ of $r=3$



$A = \frac{1}{2}(2x)(3-y)$ $\quad x^2+y^2=9$

$A = x(3-y)$ $\quad x = \sqrt{9-y^2}$

$A = (3-y)\sqrt{9-y^2}$

$A' = (3-y)\cdot \frac{1}{2}(9-y^2)^{-\frac{1}{2}}\cdot -2y$

$+ -\sqrt{9-y^2}$

$A' = \frac{-3y+y^2}{\sqrt{9-y^2}} - \sqrt{9-y^2}\left(\frac{\sqrt{9-y^2}}{\sqrt{9-y^2}}\right)$

$0 = \frac{-3y+y^2-9+y^2}{\sqrt{9-y^2}}$

$0 = 2y^2-3y-9$

$y = (2y+3)(y-3)$

$y = -\frac{3}{2}, \cancel{3}$

$A = (3-y)\sqrt{9-y^2}$

$A = (3+\frac{3}{2})\sqrt{9-(\frac{3}{2})^2}$

$A = \frac{9}{2}\sqrt{9-\frac{9}{4}}$

$A = \frac{9}{2}\sqrt{\frac{27}{4}}$

$A = \frac{9}{2}\cdot\frac{3\sqrt{3}}{2}$

$$\boxed{A = \frac{27\sqrt{3}}{4}}$$