$f'(e)$ for $f(x) = (\ln x)^x$

$$y = (\ln x)^x$$

$$\ln y = \ln (\ln x)^x$$

$$\ln y = x \ln(\ln x)$$

$$\frac{d}{dx} \ln(\ln x) = \underbrace{\frac{1}{\ln x}} \cdot \frac{1}{x}$$

$$\frac{1}{y}\frac{dy}{dx} = \cancel{x} \cdot \underbrace{\frac{1}{\ln x} \cdot \frac{1}{\cancel{x}}} + \ln(\ln x)$$

$$\frac{dy}{dx} = y\left[\frac{1}{\ln x} + \ln(\ln x)\right]$$

$$\frac{dy}{dx} = (\ln x)^x \left[\frac{1}{\ln x} + \ln(\ln x)\right]$$

$$\left.\frac{dy}{dx}\right|_{x=e} = (\ln e)^e \left[\frac{1}{\ln e} + \ln(\ln e)\right]$$

$$= 1^e \cdot \left[\frac{1}{1} + \ln(1)\right]$$

$$= 1 \cdot [1 + 0]$$

$$= 1$$

Homework 10

1) $\frac{d}{dx}\left(15\sin(-\pi x)\tan\left(\frac{2}{3}\pi x\right)\right) =$

$15\sin(-\pi x)\sec^2\left(\frac{2}{3}\pi x\right)\cdot\frac{2}{3}\pi + \tan\left(\frac{2}{3}\pi x\right)\cdot 15\cos(-\pi x)\cdot -\pi$

$= 10\pi\sin(-\pi x)\sec^2\left(\frac{2}{3}\pi x\right) - 15\pi\tan\left(\frac{2}{3}\pi x\right)\cos(-\pi x)$

2) $\frac{d}{dx}\left(-15\sin\left(\frac{5}{2}\pi x\right)\sin\left(\frac{1}{3}\pi x\right)\right) =$

$-15\sin\left(\frac{5\pi}{2}x\right)\cos\left(\frac{\pi}{3}x\right)\cdot\frac{\pi}{3} + \sin\left(\frac{\pi}{3}x\right)\cdot -15\cos\left(\frac{5\pi}{2}x\right)\cdot\frac{5\pi}{2}$

$= -5\pi\sin\left(\frac{5\pi}{2}x\right)\cos\left(\frac{\pi}{3}x\right) - \frac{75\pi}{2}\sin\left(\frac{\pi}{3}x\right)\cos\left(\frac{5\pi}{2}x\right)$

$f(x) = \sin^2(2x) = (\sin(2x))^2$

$f'(x) = 2\sin(2x)\cdot\cos(2x)\cdot 2$

$f(x) = \sin(2x)^2 = \cos(2x)^2$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\csc\theta = \frac{1}{\sin\theta}$

3) $\frac{d}{dx}\left(-e^{(3x-4)}\right)$

$= -e^{(3x-4)}\cdot 3$

$= -3e^{(3x-4)}$