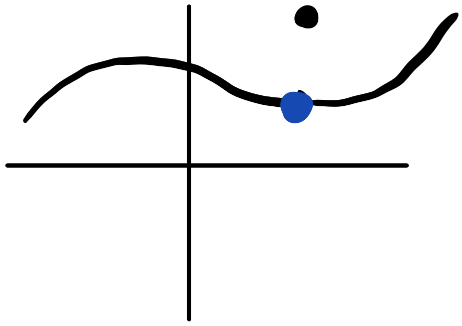
$$f(x) = \begin{cases} 3x - \frac{3}{2} & x \le 1 \\ \dfrac{x^2+3x-10}{x^2+x-6} & x > 1 \end{cases}$$

continuous @ $x=1$?



1) $f(1)$ defined

2) $\lim_{x\to 1} f(x)$ exists

3) $\lim_{x\to 1} f(x) = f(1)$

Homework 6

4) $f(x) = \dfrac{3}{\sqrt{x}}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{3}{\sqrt{x+h}} - \dfrac{3}{\sqrt{x}}}{h}$$

$$= \lim_{h\to 0} \frac{3\sqrt{x} - 3\sqrt{x+h}}{h\sqrt{x}\cdot\sqrt{x+h}} \left(\frac{3\sqrt{x} + 3\sqrt{x+h}}{3\sqrt{x} + 3\sqrt{x+h}} \right)$$

$$= \lim_{h \to 0} \frac{9x - 9(x+h)}{h\sqrt{x}\cdot\sqrt{x+h}\,(3\sqrt{x}+3\sqrt{x+h})}$$

$$= \lim_{h \to 0} \frac{\cancel{9x} - \cancel{9x} - 9\cancel{h}}{\cancel{h}\sqrt{x}\cdot\sqrt{x+h}\,(3\sqrt{x}+3\sqrt{x+h})}$$

$$= \lim_{h \to 0} \frac{-9}{\sqrt{x}\cdot\sqrt{x+h}\,(3\sqrt{x}+3\sqrt{x+h})}$$

$$= \frac{-9}{\sqrt{x}\cdot\sqrt{x}\,(3\sqrt{x}+3\sqrt{x})} = \frac{-9}{x(6\sqrt{x})}$$

$$= \frac{-9}{6x^{3/2}} = \frac{-3}{2x^{3/2}}$$

check:

$$f(x) = \frac{3}{\sqrt{x}} = 3x^{-\frac{1}{2}}$$

$$= -\frac{3}{2}x^{-\frac{3}{2}}$$

$$= -\frac{3}{2x^{3/2}}$$ ✓

$$\frac{d}{dx}(e^x) = ?$$

$$\frac{d}{dx}(a^x) = \ln a \cdot a^x$$

$$\frac{d}{dx}(e^x) = \ln e \cdot e^x = 1 \cdot e^x = e^x$$

1) $f(x) = 7x^3 - 4x^2 + 6x - 1$

$f'(x) = 21x^2 - 8x + 6$

$f''(x) = 42x - 8$

$f'''(x) = 42$

$f^{(4)}(x) = 0$

2) $f(x) = x^{\frac{5}{2}} - 3x^{-\frac{2}{3}}$

$f'(x) = \frac{5}{2}x^{\frac{3}{2}} + 2x^{-\frac{5}{3}}$

3) $f(x) = \dfrac{4}{\sqrt[4]{x}} = 4x^{-\frac{1}{4}}$

$f'(x) = -x^{-\frac{5}{4}}$

4) $f(x) = (x^2 - 1)(x + 4)$

$f'(x) = (x^2 - 1) \cdot 1 + (x + 4)(2x)$

$= (x^2 - 1) + 2x(x + 4)$

$= 3x^2 + 8x - 1$

$f(x) = x^3 + 4x^2 - x - 4$

$f'(x) = 3x^2 + 8x - 1$

5) $f(x) = (4x^5 - 7x^4 + 8x^3 - 6x^2 + 9x - 7)(x^2 - 2x + 1)$

$f'(x) = (4x^5 - 7x^4 + 8x^3 - 6x^2 + 9x - 7)(2x - 2)$

$+ (x^2 - 2x + 1)(20x^4 - 28x^3 + 24x^2 - 12x + 9)$

6) $f(x) = \dfrac{e^x - 2x}{x^2}$

$f'(x) = \dfrac{x^2(e^x - 2) - (e^x - 2x)(2x)}{x^4}$

7) $f(x) = \dfrac{x^2 e^x}{(x-1)}$

$f'(x) = \dfrac{(x-1)(x^2 e^x + e^x \cdot 2x) - x^2 e^x \cdot 1}{(x-1)^2}$

8) $f(x) = \dfrac{\frac{e^x}{\sqrt{x}}}{(x^2 - 2)} = \dfrac{e^x \cdot x^{-\frac{1}{2}}}{(x^2 - 2)}$

$$f'(x) = \frac{(x^2-2)\left(e^x \cdot -\frac{1}{2}x^{-\frac{3}{2}} + x^{-\frac{1}{2}}e^x\right) - e^x x^{-\frac{1}{2}}(2x)}{(x^2-2)^2}$$

$$= \frac{(x^2-2)\left(\frac{-e^x}{2x^{3/2}} + \frac{e^x}{\sqrt{x}}\right) - \frac{2xe^x}{\sqrt{x}}}{(x^2-2)^2}$$

$$= \frac{(x^2-2)\left(\frac{-e^x}{2x^{3/2}} + \frac{e^x}{\sqrt{x}}\right) - 2\sqrt{x}e^x}{(x^2-2)^2}$$

$$f(x) = \left(x^2 + \frac{xe^x}{x-1} + \sqrt{x}\right)\left(\frac{(x^2-4)}{e^x} + 1\right)$$

$$f'(x) = \left(x^2 + \frac{xe^x}{x-1} + \sqrt{x}\right)\left(\frac{e^x \cdot 2x - (x^2-4)e^x}{e^{2x}}\right) +$$

$$\left(\frac{x^2-4}{e^x} + 1\right)\left(2x + \frac{(x-1)(xe^x + e^x) - x^2e^x}{(x-1)^2} + \frac{1}{2}x^{-\frac{1}{2}}\right)$$