$\int u\,dv = uv - \int v\,du$

6) $\int t^2 \underbrace{\sqrt{t-2}}\,dt$

$u = t^2 \qquad v = \frac{2}{3}(t-2)^{\frac{3}{2}}$

$du = 2t\,dt \quad dv = \sqrt{t-2}\,dt$

$= \frac{2}{3}t^2(t-2)^{\frac{3}{2}} - \int \frac{4}{3}t(t-2)^{\frac{3}{2}}\,dt$

$u = t-2 \Rightarrow t = u+2$

$du = dt$

$= \frac{2}{3}t^2(t-2)^{\frac{3}{2}} - \frac{4}{3}\int u^{\frac{3}{2}}(u+2)\,du$

$= \frac{2}{3}t^2(t-2)^{\frac{3}{2}} - \frac{4}{3}\int u^{\frac{5}{2}} + 2u^{\frac{3}{2}}\,du$

$= \frac{2}{3}t^2(t-2)^{\frac{3}{2}} - \frac{4}{3}\left[\frac{2}{7}u^{\frac{7}{2}} + 2\cdot\frac{2}{5}u^{\frac{5}{2}}\right] + C$

$= \frac{2}{3}t^2(t-2)^{\frac{3}{2}} - \frac{8}{21}(t-2)^{\frac{7}{2}} - \frac{16}{15}(t-2)^{\frac{5}{2}} + C$

7) $\int e^{-2x}\sin(2x)\,dx = -\frac{1}{2}e^{-2x}\sin(2x) + \int e^{-2x}\cos(2x)\,dx$

$u = \sin(2x) \qquad v = -\frac{1}{2}e^{-2x}$

$du = 2\cos(2x)\,dx \quad dv = e^{-2x}\,dx$

$\begin{cases} u = \cos(2x) & v = -\frac{1}{2}e^{-2x} \\ du = -2\sin(2x)\,dx & dv = e^{-2x}\,dx \end{cases}$

$\int e^{-2x}\sin(2x)\,dx = -\frac{1}{2}e^{-2x}\sin(2x) - \frac{1}{2}e^{-2x}\cos(2x) - \int e^{-2x}\sin(2x)\,dx$

$$2\int e^{-2x}\sin(2x)dx = -\frac{1}{2}e^{-2x}\sin(2x) - \frac{1}{2}e^{-2x}\cos(2x) + C$$

$$\int e^{-2x}\sin(2x)dx = -\frac{1}{4}e^{-2x}\sin(2x) - \frac{1}{4}e^{-2x}\cos(2x) + C$$

9) $\int \sin(\ln x)\,dx = x\sin(\ln x) - \int \cos(\ln x)dx$

$u = \sin(\ln x)$ $\quad v = x$

$du = \frac{1}{x}\cos(\ln x)dx$ $\quad dv = dx$

$u = \cos(\ln x)$ $\quad v = x$

$du = \frac{-1}{x}\sin(\ln x)dx$ $\quad dv = dx$

$$\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x) + \int -\sin(\ln x)\,dx$$

$$2\int \sin(\ln x)\,dx = x\sin(\ln x) - x\cos(\ln x) + C$$

$$\int \sin(\ln x)\,dx = \frac{x}{2}(\sin(\ln x - \cos(\ln x)) + C$$

10) $\int_{-1}^{1} \arctan x\,dx = x\arctan x - \int_{-1}^{1} \frac{x}{1+x^2}dx$

$u = \arctan x$ $\quad v = x$

$du = \frac{1}{1+x^2}dx$ $\quad dv = dx$

$u = 1+x^2$

$du = 2x\,dx$

$$= x\arctan x - \frac{1}{2}\int \frac{1}{u}du$$

$$= x \arctan x - \frac{1}{2} \ln|u| \Big|$$

$$= x \arctan x - \frac{1}{2} \ln(1+x^2) \Big|_{-1}^{1}$$

$$= \arctan(1) - \frac{1}{2} \ln(2) - \left[-\arctan(-1) - \frac{1}{2} \ln(2) \right]$$

$$= \frac{\pi}{4} - \frac{1}{2} \ln(2) + -\frac{\pi}{4} + \frac{1}{2} \ln(2)$$

$$= 0$$