$$\sin^2 x = \tfrac{1}{2} - \tfrac{1}{2}\cos(2x)$$

$$\cos^2 x = \tfrac{1}{2} + \tfrac{1}{2}\cos(2x)$$

$$\int \sin^2 x \cos^4 x \, dx = \int \left(\tfrac{1}{2} - \tfrac{1}{2}\cos(2x)\right)\left(\tfrac{1}{2} + \tfrac{1}{2}\cos(2x)\right)^2 dx$$

$$\int \left(\tfrac{1}{2} - \tfrac{1}{2}\cos(2x)\right)\left(\tfrac{1}{4} + \tfrac{1}{2}\cos(2x) + \tfrac{1}{4}\cos^2(2x)\right) dx$$

$$\int \cancel{\tfrac{1}{8}} + \tfrac{1}{4}\cancel{\cos(2x)} + \tfrac{1}{8}\cancel{\cos^2(2x)} - \tfrac{1}{8}\cancel{\cos(2x)} - \tfrac{1}{4}\cancel{\cos^2(2x)} - \tfrac{1}{8}\cos^3(2x) \, dx$$

$$\int \tfrac{1}{8} + \tfrac{1}{8}\cos(2x) - \tfrac{1}{8}\cos^2(2x) - \tfrac{1}{8}\cos^3(2x) \, dx$$

$$\cos^2(2x)\cos(2x)$$

$$\int \tfrac{1}{8} + \tfrac{1}{8}\cos(2x) - \tfrac{1}{8}\left[\tfrac{1}{2} + \tfrac{1}{2}\cos(4x)\right] dx - \tfrac{1}{8}\int \cos^3(2x)\,dx$$

$$\int \tfrac{1}{8} + \tfrac{1}{8}\cos(2x) - \tfrac{1}{16} - \tfrac{1}{16}\cos(4x)\,dx - \tfrac{1}{8}\int (1-\sin^2(2x))\cos(2x)\,dx$$

$$\int \tfrac{1}{16} + \tfrac{1}{8}\cos(2x) - \tfrac{1}{16}\cos(4x)\,dx - \tfrac{1}{8}\int \cos(2x) - \sin^2(2x)\cdot\cos(2x)\,dx$$

$$\tfrac{1}{16}x - \tfrac{1}{16}\sin(2x) - \tfrac{1}{64}\sin(4x) - \tfrac{1}{8}\int \cos(2x) + \tfrac{1}{8}\int \sin^2(2x)\cdot\cos(2x)\,dx$$

$$u = \sin(2x)$$

$$du = 2\cos(2x)\,dx$$

$$\tfrac{1}{16}x - \tfrac{1}{16}\sin(2x) - \tfrac{1}{64}\sin(4x) - \tfrac{1}{16}\sin(2x) + \tfrac{1}{8}\int \tfrac{1}{2}u^2\,du$$

$$\tfrac{1}{16}x - \tfrac{1}{16}\sin(2x) - \tfrac{1}{64}\sin(4x) - \tfrac{1}{16}\sin(2x) + \tfrac{1}{16}\cdot\tfrac{1}{3}u^3 + C$$

$$\tfrac{1}{16}x - \tfrac{1}{16}\sin(2x) - \tfrac{1}{64}\sin(4x) - \tfrac{1}{16}\sin(2x) + \tfrac{1}{48}\sin^3(2x) + C$$

7) $\int \tan^5 x\, dx = \int \tan^2 x \cdot \tan^3 x\, dx$

$= \int (\sec^2 x - 1)\tan^3 x\, dx$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

$\tan x \cdot \tan^2 x$

$= \int \sec^2 x \tan^3 x - \tan^3 x\, dx$

$= \int \sec^2 x \tan^3 x - \tan x(\sec^2 x - 1)\, dx$

$= \int \sec^2 x \tan^3 x - \tan x \sec^2 x + \tan x\, dx$

$\frac{\sin x}{\cos x}$

$= \int \sec^2 x \tan^3 x\, dx - \int \tan x \sec^2 x\, dx + \int \tan x\, dx$

$u_1 = \tan x$ $\quad u_2 = \tan x$ $\quad u_3 = \cos x$

$du_1 = \sec^2 x\, dx$ $\quad du_2 = \sec^2 x\, dx$ $\quad du_3 = -\sin x\, dx$

$= \int u_1^3\, du_1 - \int u_2\, du_2 - \int \frac{1}{u_3}\, du_3$

$= \frac{1}{4}u_1^4 - \frac{1}{2}u_2^2 - \ln|u_3| + C$

$= \frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x - \ln|\cos x| + C$