6) $\int_0^{\pi} \sin^4(x)\,dx$

$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x)$

$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos(2x)$

$= \int_0^{\pi} (\sin^2 x)^2\,dx = \int_0^{\pi} \left(\frac{1}{2} - \frac{1}{2}\cos(2x)\right)^2 dx = \int_0^{\pi} \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\cos^2(2x)\,dx$

$= \int_0^{\pi} \frac{1}{4} - \frac{1}{2}\cos(2x) + \frac{1}{4}\left[\frac{1}{2} + \frac{1}{2}\cos(4x)\right] dx$

$= \frac{1}{4}x - \frac{1}{4}\sin(2x) + \frac{1}{8}x + \frac{1}{32}\sin(4x)\Big|_0^{\pi}$

$= \frac{\pi}{4} - \frac{1}{4}\cancel{\sin(2\pi)} + \frac{\pi}{8} + \frac{1}{32}\cancel{\sin(4\pi)} - \cancel{0}\cancel{4}\cancel{\sin 0} - \cancel{0} - \cancel{\sin 0}$

$= \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$

4) $\int_0^{\frac{\pi}{4}} x\sec x\tan x\,dx = x\sec x - \int_0^{\frac{\pi}{4}} \sec x\,dx$

$\int \sec x\,dx = \ln|\sec x + \tan x|$

$u = x$ $\quad v = \sec x$

$du = dx$ $\quad dv = \sec x\tan x$

$= x\sec x - \ln|\sec x + \tan x|\Big]_0^{\frac{\pi}{4}}$

$= \frac{\pi}{4}\sec\frac{\pi}{4} - \ln\left|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right| - \left[0 - \ln|\sec 0 + \tan 0|\right]$

$= \frac{\pi}{4}\sqrt{2} - \ln|\sqrt{2} + 1| + \ln|1|$

$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$= \frac{\pi\sqrt{2}}{4} - \ln(1 + \sqrt{2})$

9) $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos^2 x - \sin^2 x \, dx$

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

$$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos^2 x - (1 - \cos^2 x) \, dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} 2\cos^2 x - 1 \, dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} 1 + \cos(2x) - 1 \, dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \cos(2x) \, dx = \frac{1}{2} \sin(2x) \Big|_{\frac{\pi}{8}}^{\frac{\pi}{4}}$$

$$= \frac{1}{2} \sin\left(\frac{\pi}{2}\right) - \frac{1}{2} \sin\left(\frac{\pi}{4}\right) = \frac{1}{2} - \frac{\sqrt{2}}{4}$$

7) $\int \tan^5 x \, dx = \int \tan^2 x \cdot \tan^3 x \, dx = \int (\sec^2 x - 1) \tan^3 x \, dx$

$$= \int \sec^2 x \tan^3 x - \tan^3 x \, dx = \int \sec^2 x \tan^3 x \, dx - \int \tan^3 x \, dx$$

$$= \int \sec^2 x \tan^3 x \, dx - \int (\sec^2 x - 1) \tan x \, dx$$

$$= \int \sec^2 x \tan^3 x \, dx - \int \sec^2 x \tan x \, dx + \int \tan x \, dx$$

$$= \int \sec^2 x \tan^3 x \, dx - \int \sec^2 x \tan x \, dx + \int \frac{\sin x}{\cos x} \, dx$$

$u_1 = \tan x$ $\quad$ $u_2 = \tan x$ $\quad$ $u_3 = \cos x$

$du_1 = \sec^2 x \, dx$ $\quad$ $du_2 = \sec^2 x \, dx$ $\quad$ $du_3 = -\sin x \, dx$

$$= \int u_1^3 \, du_1 - \int u_2 \, du_2 - \int \frac{1}{u_3} \, du_3$$

$$= \frac{1}{4} u_1^4 - \frac{1}{2} u_2^2 - \ln|u_3| + C$$

$$= \frac{1}{4} \tan^4 x - \frac{1}{2} \tan^2 x - \ln|\cos x| + C$$