5. Consider the following table of expressions for functions:

$\cancel{\frac{\sin(x)}{x}}$	$\cancel{x^2\cos\left(\frac{1}{x}\right)}$	$\cancel{\ln(x^2)}$
$\cancel{\frac{1-\cos(x)}{x}}$	$\cancel{\sin\left(\frac{1}{x}\right)}$	$\cancel{\frac{1}{x}}$
$\cancel{\arctan(x)}$	$\cancel{e^{1/x}}$	$\cancel{\frac{x}{\lvert x\rvert}}$

Use the table above to give examples of each of the following. Only list one function for each part, even if there are multiple functions that have the desired property.

(a) A function $f(x)$ where $\lim_{x\to 0} f(x) = 0$

$\lim_{x\to 0} \arctan x = 0$

$-1 \le \cos\left(\frac{1}{x}\right) \le 1$

$-x^2 \le x^2\cos\left(\frac{1}{x}\right) \le x^2$

$0 \quad \lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right) = 0 \quad 0$

(b) A function $f(x)$ where $\lim_{x\to 0} f(x) = 1$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{1-\cos(x)}{x}\left(\frac{1+\cos(x)}{1+\cos(x)}\right)$

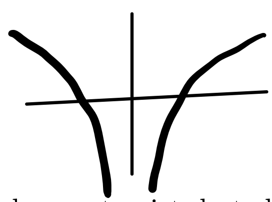
$= \lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)}$

$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1+\cos x} = 1 \cdot \frac{0}{2}$

$= 0$

(c) A function $f(x)$ where $\lim_{x\to 0} f(x) = \infty$ or $-\infty$

$\lim_{x\to 0} \ln(x^2) = -\infty$



(d) A function $f(x)$ where $\lim_{x\to 0} f(x)$ does not exist, but $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$ both exist (either finite or infinite)

$\lim_{x\to 0^-} \frac{1}{x} = -\infty$

$\lim_{x\to 0^+} \frac{1}{x} = \infty$ DNE

$\lim_{x\to 0^-} e^{\frac{1}{x}} = 0$

$\lim_{x\to 0^+} e^{\frac{1}{x}} = \infty$ DNE

$\frac{x}{|x|} = \begin{cases} \frac{-x}{x} & x < 0 \\ \frac{x}{x} & x > 0 \end{cases}$

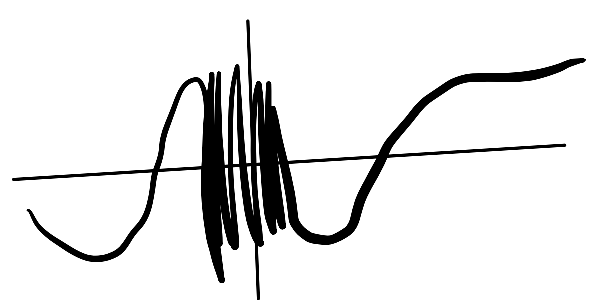
$\lim_{x\to 0^-} -1 = -1 \quad \lim_{x\to 0^+} 1 = 1$

DNE

(e) A function $f(x)$ where neither $\lim_{x\to 0^-} f(x)$ nor $\lim_{x\to 0^+} f(x)$ exists (Neither finite nor infinite)

$\lim_{x\to 0^-} \sin\frac{1}{x} = $ DNE

$\lim_{x\to 0^+} \sin\frac{1}{x} = $ DNE



Homework 3

6) $$\lim_{x\to 4} \frac{3(\ln(9x+10)+1)}{4((2x+2)^{\frac{1}{3}}+1)} = \frac{3(\ln(36+10)+1)}{4((8+2)^{\frac{1}{3}}+1)}$$

$$= \frac{3(\ln 46+1)}{4\sqrt[3]{10}+1}$$

$$|x| = \begin{cases} -x & x<0 \\ x & x\geq 0 \end{cases}$$

13) $$\lim_{x\to -5} \frac{-8|x|+40}{2x+10}$$

$$= \lim_{x\to -5} \frac{-8(-x)+40}{2x+10} = \lim_{x\to -5} \frac{8x+40}{2x+10} = \lim_{x\to -5} \frac{8\cancel{(x+5)}}{2\cancel{(x+5)}} = 4$$