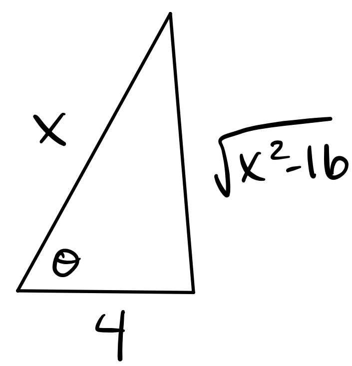
$$\int \frac{1}{x^2\sqrt{x^2-16}}\,dx \qquad x = 4\sec\theta$$
$$dx = 4\sec\theta\tan\theta\,d\theta$$

$$\int \frac{4\sec\theta\tan\theta\,d\theta}{16\sec^2\theta\sqrt{16\sec^2\theta-16}} = \frac{1}{4}\int \frac{\sec\theta\tan\theta\,d\theta}{\sec^2\theta\cdot 4\sqrt{\sec^2\theta-1}}$$

$$= \frac{1}{16}\int \frac{\sec\theta\tan\theta\,d\theta}{\sec^2\theta\sqrt{\tan^2\theta}} = \frac{1}{16}\int \frac{\sec\theta\tan\theta}{\sec^2\theta\tan\theta}\,d\theta$$

$$\sec\theta = \frac{x}{4}$$



$$= \frac{1}{16}\int \cos\theta\,d\theta = \frac{1}{16}\sin\theta + C$$

$$= \frac{1}{16}\left(\frac{\sqrt{x^2-16}}{x}\right) + C$$

5) $$\int \sqrt{x^2-1}\,dx \qquad x = \sec\theta$$
$$dx = \sec\theta\tan\theta\,d\theta$$

$$\int \sqrt{\sec^2\theta-1}\cdot\sec\theta\tan\theta\,d\theta = \int \tan\theta\cdot\sec\theta\cdot\tan\theta\,d\theta$$

$$= \int \tan^2\theta\cdot\sec\theta\,d\theta = \int (\sec^2\theta-1)\sec\theta\,d\theta$$

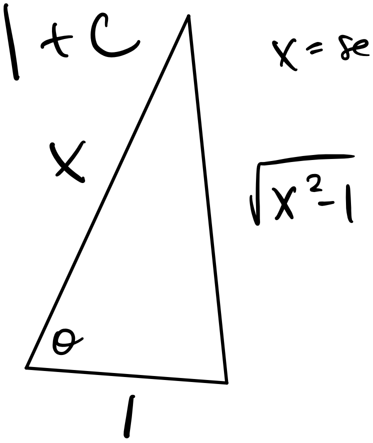
$$= \int \sec^3\theta - \sec\theta\,d\theta$$

$$= \frac{1}{2}\left(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right) - \ln|\sec\theta + \tan\theta| + C$$

$$= \frac{1}{2}\left(x\sqrt{x^2-1} + \ln\left|x+\sqrt{x^2-1}\right|\right) - \ln\left|x+\sqrt{x^2-1}\right| + C$$

$$= \frac{1}{2}x\sqrt{x^2-1} - \frac{1}{2}\ln\left|x+\sqrt{x^2-1}\right| + C$$

$x = \sec\theta$



5) $$\int \frac{x^2+1}{(x^2+2x+3)^2}\,dx = \frac{Ax+B}{x^2+2x+3} + \frac{Cx+D}{(x^2+2x+3)^2}$$

$$x^2+1 = (Ax+B)(x^2+2x+3) + Cx + D$$

$$x^2+1 = \cancel{Ax^3} + \cancel{2Ax^2} + 3\cancel{Ax} + \cancel{Bx^2} + 2\cancel{Bx} + 3B + \cancel{Cx} + D$$

$$\underbrace{0}x^3 + \underbrace{1}x^2 + \underbrace{0}x + \underbrace{1}$$

$$x^2+1 = \underbrace{(A)}x^3 + \underbrace{(2A+B)}x^2 + \underbrace{(3A+2B+C)}x + \underbrace{(3B+D)}$$

$A = 0$

$1 = 2A + B$

$1 = 2\cdot 0 + B$

$B = 1$

$3A + 2B + C = 0$

$0 + 2 + C = 0$

$C = -2$

$3B + D = 1$

$3\cdot 1 + D = 1$

$3 + D = 1$

$D = -2$

$$= \int \frac{1}{x^2+2x+3} + \frac{-2x-2}{(x^2+2x+3)^2}\,dx$$

$$= \int \frac{1}{(x+1)^2+2}\,dx - \int \frac{2x+2}{(x^2+2x+3)^2}\,dx$$

$x+1=\sqrt{2}\tan\theta$

$dx=\sqrt{2}\sec^2\theta\,d\theta$

$\Bigg\}$

$u=x^2+2x+3$

$du=2x+2$

$$= \int \frac{\sqrt{2}\sec^2\theta\,d\theta}{2\tan^2\theta+2} - \int \frac{1}{u^2}\,du$$

$$= \frac{1}{2}\int \frac{\sqrt{2}\sec^2\theta\,d\theta}{\sec^2\theta} + u^{-1} + C$$

$$= \frac{1}{2}\int \sqrt{2}\,d\theta + \frac{1}{x^2+2x+3} + C$$

$$= \frac{1}{2}\cdot\sqrt{2}\,\theta + \frac{1}{x^2+2x+3} + C$$

$$= \frac{\sqrt{2}}{2}\arctan\left(\frac{x+1}{\sqrt{2}}\right) + \frac{1}{x^2+2x+3} + C$$

$x+1=\sqrt{2}\tan\theta$

$\frac{x+1}{\sqrt{2}}=\tan\theta$

$\theta=\arctan\left(\frac{x+1}{\sqrt{2}}\right)$