8) $\displaystyle\int \frac{dy}{y^2\sqrt{y^2+4}}$ $\qquad y = 2\tan\theta$

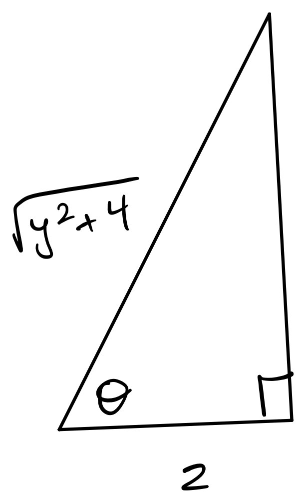
$\qquad\qquad\qquad\qquad dy = 2\sec^2\theta \, d\theta$

$$= \int \frac{2\sec^2\theta \, d\theta}{4\tan^2\theta\sqrt{4\tan^2\theta+4}} = \frac{1}{2}\int \frac{\sec^2\theta \, d\theta}{\tan^2\theta \cdot 2\sqrt{\tan^2\theta+1}} = \frac{1}{4}\int \frac{\sec^2\theta \, d\theta}{\tan^2\theta \cdot \sec\theta}$$

$$= \frac{1}{4}\int \frac{\sec\theta \, d\theta}{\tan^2\theta} = \frac{1}{4}\int \frac{\cos^2\theta}{\sin^2\theta\cos\theta} \, d\theta = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta} \, d\theta$$

$u = \sin\theta \quad du = \cos\theta \, d\theta$

$$= \frac{1}{4}\int \frac{1}{u^2} \, du = \frac{1}{4} \cdot -\frac{1}{u} + C = -\frac{1}{4}\csc\theta + C$$



$y = 2\tan\theta$

$\tan\theta = \frac{y}{2}$

$$= -\frac{1}{4} \cdot \frac{\sqrt{y^2+4}}{y} + C$$

$$= -\frac{\sqrt{y^2+4}}{4y} + C$$

6) $\displaystyle\int_0^{1/6} \frac{1}{(1-9x^2)^2}\,dx$

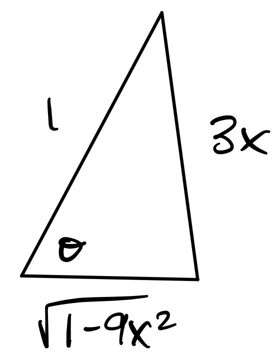
$3x = \sin\theta$

$x = \frac{1}{3}\sin\theta$

$dx = \frac{1}{3}\cos\theta\,d\theta$

$$= \int \frac{1}{(1-\sin^2\theta)^2}\cdot\frac{1}{3}\cos\theta\,d\theta = \frac{1}{3}\int\frac{\cos\theta}{\cos^4\theta}\,d\theta$$

$$= \frac{1}{3}\int \sec^3\theta\,d\theta = \frac{1}{3}\cdot\frac{1}{2}\left(\sec\theta\cdot\tan\theta + \ln|\sec\theta + \tan\theta|\right)$$



$$= \frac{1}{6}\left[\frac{1}{\sqrt{1-9x^2}}\cdot\frac{3x}{\sqrt{1-9x^2}} + \ln\left|\frac{1}{\sqrt{1-9x^2}} + \frac{3x}{\sqrt{1-9x^2}}\right|\right]_0^{1/6}$$

$$= \frac{1}{6}\left[\frac{3x}{1-9x^2} + \ln\left|\frac{1+3x}{\sqrt{1-9x^2}}\right|\right]_0^{1/6}$$

$$= \frac{1}{6}\left[\frac{3\cdot\frac{1}{6}}{1-9\cdot\frac{1}{36}} + \ln\left|\frac{1+\frac{1}{2}}{\sqrt{1-9\cdot\frac{1}{36}}}\right| - 0 - \cancel{\ln 1}\right]$$

$$= \frac{1}{6}\left[\frac{\frac{1}{2}}{1-\frac{1}{4}} + \ln\left|\frac{\frac{3}{2}}{\sqrt{1-\frac{1}{4}}}\right|\right] = \frac{1}{6}\left[\frac{\frac{1}{2}}{\frac{3}{4}} + \ln\left|\frac{\frac{3}{2}}{\sqrt{\frac{3}{4}}}\right|\right]$$

$$= \frac{1}{6}\left[\frac{2}{3} + \ln\left|\frac{3}{2}\left(\frac{3}{4}\right)^{-\frac{1}{2}}\right|\right]$$

$$\int \frac{x^2-5x-9}{(x-1)^3}\,dx \Rightarrow \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

$$x^2-5x-9 = A(x-1)^2 + B(x-1) + C$$

$$x^2-5x-9 = A(x^2-2x+1) + Bx - B + C$$

$$x^2-5x-9 = A\cancel{x}^2 - 2\cancel{Ax} \cancel{+A} + \cancel{Bx} - \cancel{B} + \cancel{C}$$

$$\underline{x}^2 - \underline{5}x - \underline{9} = \underline{(A)}x^2 + \underline{(-2A+B)}x + (\underline{A-B+C})$$

$$A = 1$$

$$-2A + B = -5$$
$$-2 + B = -5$$
$$B = -3$$

$$A - B + C = -9$$
$$1 + 3 + C = -9$$
$$4 + C = -9$$
$$C = -13$$

$$\int \frac{1}{x-1} + \frac{-3}{(x-1)^2} + \frac{-13}{(x-1)^3}\,dx$$

$$= \ln|x-1| + 3\frac{1}{(x-1)} + \frac{+13}{2(x-1)^2} + C$$

$$= \ln|x-1| + \frac{3}{x-1} + \frac{13}{2(x-1)^2} + C$$

$$\int \frac{dx}{s^2(s-1)^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} + \frac{D}{(s-1)^2}$$

$$1 = s(s-1)^2 A + (s-1)^2 B + s^2(s-1)C + s^2 D$$