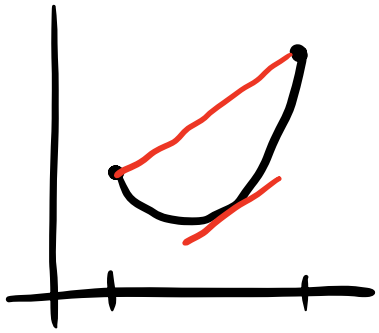
Homework 17



12: $f(x) = \sqrt{x} + 2 \quad [4, 9]$

$f(9) = \sqrt{9} + 2 = 3 + 2 = 5$

$f(4) = \sqrt{4} + 2 = 2 + 2 = 4$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$\frac{5-4}{9-4} = \frac{1}{5}$

$\frac{1}{5} = \frac{1}{2\sqrt{c}}$

$2\sqrt{c} = 5$

$\sqrt{c} = \frac{5}{2}$

$c = \frac{25}{4}$

11) $f(x) = 2x^2 - 3x + 1 \quad [0, 2]$

$f(0) = 1$

$f(2) = 2 \cdot 4 - 6 + 1 = 8 - 6 + 1 = 3$

$\frac{f(2) - f(0)}{2 - 0} = f'(c)$

$\frac{3-1}{2-0} = 4c - 3$

$\frac{2}{2} = 4c - 3$

$\Rightarrow$

$1 = 4c - 3$

$4c = 4$

$c = 1$

$f(b) = f(a)$

$$\frac{f(b)-f(a)}{b-a} = \frac{0}{b-a} = f'(c)$$

13) $f(1) = 10 \quad f'(x) \geq 2 \quad 1 \leq x \leq 4$

$f(4) \geq ?$

$$\frac{f(4)-f(1)}{4-1} = f'(x) \geq 2$$

$$\frac{f(4)-f(1)}{3} \geq 2$$

$$\frac{f(4)-10}{3} \geq 2$$

$$f(4)-10 \geq 6$$

$$f(4) \geq 16$$

Homework 16

5) $f(x) = x^2\sqrt{1-x^2}$

$f'(x) = x^2 \cdot \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot -2x + \sqrt{1-x^2} \cdot 2x$

$= \frac{-x^3}{\sqrt{1-x^2}} + 2x\sqrt{1-x^2}\left(\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\right)$

$0 = \frac{-x^3 + 2x(1-x^2)}{\sqrt{1-x^2}}$

$0 = -x^3 + 2x(1-x^2)$

$0 = -x^3 + 2x - 2x^3$

$0 = -3x^3 + 2x$

$0 = x(-3x^2 + 2)$

$x = 0$ $\quad -3x^2 + 2 = 0$

$3x^2 = 2$

$x^2 = \frac{2}{3}$

$x = \pm\sqrt{\frac{2}{3}}$

$0 = \sqrt{1-x^2}$ $\quad x^2 = 1$

$0 = 1 - x^2$

$x^2 = 1$

$x = \pm 1$

6) $f(x) = \cos(x) + \sin^2(x) \quad [0, 2\pi]$

$f'(x) = -\sin(x) + 2\sin(x)\cos(x)$

$0 = \sin(x)(-1 + 2\cos(x))$

$\sin x = 0$

$x = 0, \pi, 2\pi$

$-1 + 2\cos x = 0$

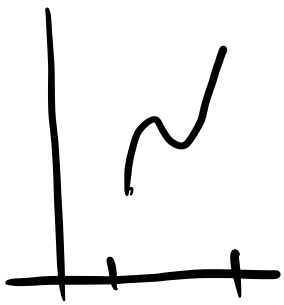
$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

8) $f(x) = \frac{\ln x}{x} \quad [1, 3]$

$f'(x) = \frac{x \cdot \frac{1}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$



$1 - \ln x = 0$

$\ln x = 1$

$x = e$

$f(1) = \frac{\ln 1}{1} = 0$ ← min

$f(e) = \frac{\ln e}{e} = \frac{1}{e}$ ← max

$f(3) = \frac{\ln 3}{3}$ ←