Homework 20

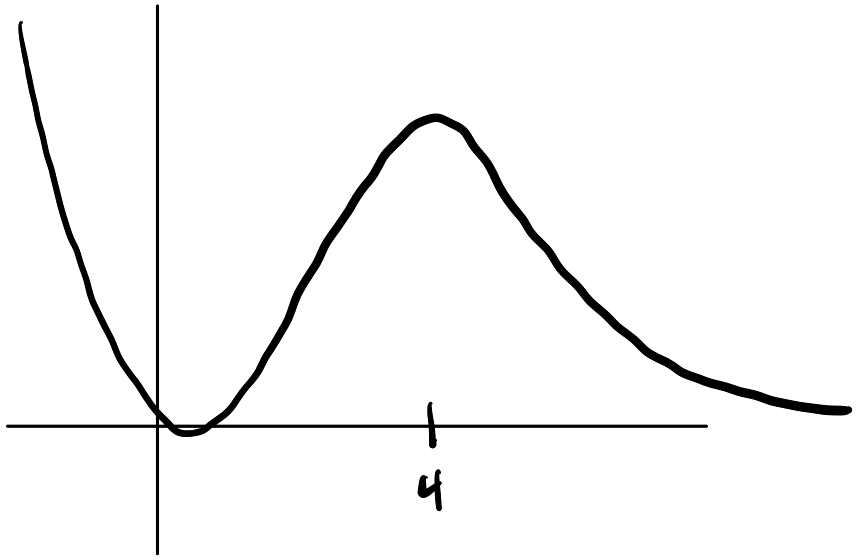
3) $f(x) = x^4e^{-x} = \frac{x^4}{e^x}$

domain: $(-\infty, \infty)$

no vertical asymptotes

horizontal asymptote at $y=0$

— dec —|— inc ——|— dec —

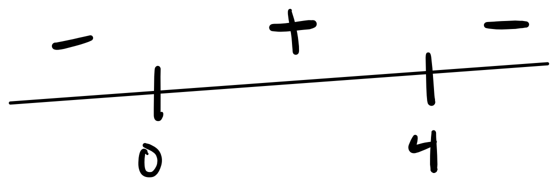


$f'(x) = x^4 \cdot -e^{-x} + e^{-x} \cdot 4x^3$

$0 = -x^4e^{-x} + 4x^3e^{-x}$

$0 = x^3\underbrace{e^{-x}}(-x+4)$

$x = 0 \qquad x = 4$



$f''(x) = -x^4 \cdot -e^{-x} + e^{-x} \cdot -4x^3$
$+ 4x^3 \cdot -e^{-x} + e^{-x} \cdot 12x^2$

$0 = x^4e^{-x} - 4x^3e^{-x}$
$- 4x^3e^{-x} + 12x^2e^{-x}$

$0 = x^4e^{-x} - 8x^3e^{-x} + 12x^2e^{-x}$

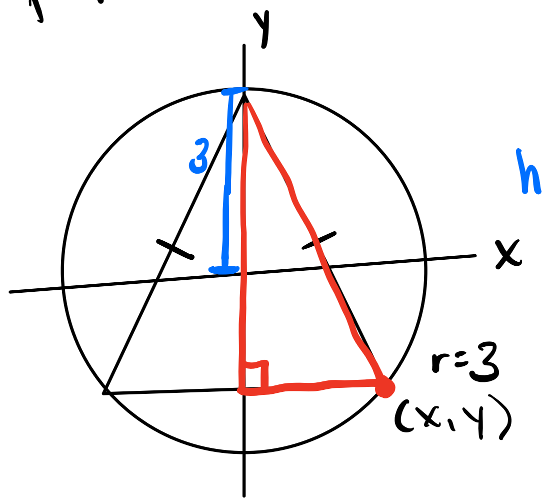
$0 = x^2e^{-x}(x^2 - 8x + 12)$

$0 = x^2e^{-x}(x-6)(x-2)$

$x = 0, 6, 2$

Homework 21

Largest area of an isosceles triangle inscribed in circle of radius 3



$A=\frac{1}{2}bh \Rightarrow A=\frac{1}{2}\cdot 2x(3-y)$

$h=3-y$

$b=2x$

$A=x(3-y)$

$A=\sqrt{9-y^2}\,(3-y)$

$A'=\sqrt{9-y^2}\,(-1)+(3-y)\cdot\cancel{\frac{1}{2}}(9-y^2)^{-\frac{1}{2}}\cdot-\cancel{2}y$

$x^2+y^2=9$

$x=\sqrt{9-y^2}$

$\left(\frac{\sqrt{9-y^2}}{\sqrt{9-y^2}}\right)\cdot \nearrow$

$A'=-\sqrt{9-y^2}-\frac{y(3-y)}{\sqrt{9-y^2}}$

$0=\frac{-(9-y^2)-y(3-y)}{\sqrt{9-y^2}}$

$y=-\frac{3}{2}$

$x=\sqrt{9-(-\frac{3}{2})^2}$

$x=\sqrt{9-\frac{9}{4}}$

$x=\sqrt{\frac{36-9}{4}}$

$x=\sqrt{\frac{27}{4}}$

$x=\frac{3\sqrt{3}}{2}$

$y=3$

$x=\sqrt{9-9}$

$\cancel{x=0}$

↑ no area

$0=-9+y^2-3y+y^2$

$0=2y^2-3y-9$

$0=(2y+3)(y-3)$

$y=-\frac{3}{2} \quad y=3$

$A=x(3-y)$

$A=\frac{3\sqrt{3}}{2}(3-{-\frac{3}{2}})$

$A=\frac{3\sqrt{3}}{2}(3+\frac{3}{2})$

$A=\frac{3\sqrt{3}}{2}\left(\frac{6+3}{2}\right)$

$$\boxed{A=\frac{3\sqrt{3}}{2}\cdot\frac{9}{2}=\frac{27\sqrt{3}}{4}}$$

2) point on $y^2=2x$ closest to $(1,4)$

$$y^2 = 2x$$
$$x = \frac{y^2}{2}$$

$$d = \sqrt{(y-y_1)^2 + (x-x_1)^2}$$

$$d = \sqrt{(y-4)^2 + (x-1)^2}$$

$$d = \sqrt{(y-4)^2 + \left(\frac{y^2}{2}-1\right)^2}$$

$$\frac{dD}{dy} = \frac{1}{2}\left((y-4)^2 + \left(\frac{y^2}{2}-1\right)^2\right)^{-\frac{1}{2}}\left[2(y-4) + 2\left(\frac{y^2}{2}-1\right)(y)\right]$$

$$0 = \frac{2y-8+y^3-2y}{2\sqrt{(y-4)^2+\left(\frac{y^2}{2}-1\right)^2}}$$

$$x = \frac{y^2}{2}$$

$$0 = y^3 - 8$$
$$y^3 = 8$$
$$y = 2$$

$$y = 2 \Rightarrow x = \frac{4}{2} = 2$$
$$y = -2 \Rightarrow x = \frac{4}{2} = 2$$

$\boxed{(2,2)}$ $(2,-2)$ $d = \sqrt{(y-4)^2 + (x-1)^2}$

$$d\Big|_{(2,2)} = \sqrt{(2-4)^2 + (2-1)^2} = \sqrt{(-2)^2+1} = \sqrt{5} \leftarrow \text{min}$$

$$d\Big|_{(2,-2)} = \sqrt{(-2-4)^2 + (2-1)^2} = \sqrt{(-6)^2+1} = \sqrt{37}$$

Homework 20

4) $f(x) = \cos^2 x - 2\sin x \quad [0, 2\pi]$

no v.a.

no h.a.

$f'(x) = -2\cos x \sin x - 2\cos x$

$0 = -2\cos x \sin x - 2\cos x$

$0 = -2\cos x(\sin x + 1)$

$-2\cos x = 0 \qquad \sin x + 1 = 0$

$\cos x = 0 \qquad \sin x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{3\pi}{2}$