Homework 18

2) $f(x) = x^4 - 2x^2 + 3$

$f'(x) = 4x^3 - 4x$

$f''(x) = 12x^2 - 4$

$0 = 4x^3 - 4x$

$0 = 4x(x^2 - 1)$

$0 = 4x(x+1)(x-1)$

$x = 0, -1, 1$

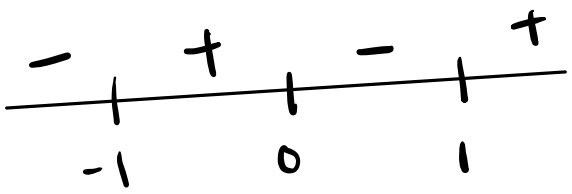
$0 = 12x^2 - 4$

$12x^2 = 4$

$x^2 = \frac{1}{3}$

$x = \pm \frac{1}{\sqrt{3}}$

$f'(x)$



increasing: $(-1,0) \cup (1,\infty)$

decreasing: $(-\infty,-1) \cup (0,1)$

local min @ $x=-1$, $x=1$

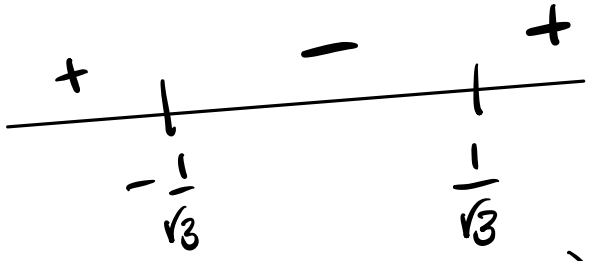
$f(-1) = 2$

$f(1) = 2$

local max @ $x=0$

$f(0) = 3$

$f''(x)$



concave up: $(-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, \infty)$

concave down: $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

inflection points? yes

$(-\frac{1}{\sqrt{3}}, \frac{22}{9})$

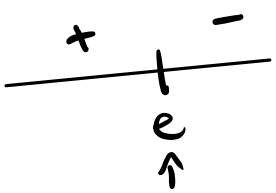
$(\frac{1}{\sqrt{3}}, \frac{22}{9})$

$[1,3]$ $x=e$

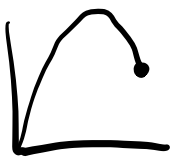
$f(1) = 0$ ← min

$f(e) = \frac{1}{e}$

$f(3) = \frac{\ln 3}{3}$



$f'(x) = \frac{1 - \ln x}{x^2}$



Homework 19

11) $\lim_{x \to 0^+} x^{\sqrt{x}}$

$y = x^{\sqrt{x}}$

$\ln y = \sqrt{x} \ln x$

$= \dfrac{\ln x}{x^{-\frac{1}{2}}}$

$\lim_{x \to 0^+} \dfrac{\ln x}{x^{-\frac{1}{2}}} = \lim_{x \to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{2}x^{-\frac{3}{2}}}$

$= \lim_{x \to 0^+} \dfrac{\frac{1}{x}}{\frac{-1}{2x^{3/2}}} = \lim_{x \to 0^+} \dfrac{1}{x} \cdot \dfrac{2x^{\frac{3}{2}}}{-1}$

$= \lim_{x \to 0^+} -2x^{\frac{1}{2}} = 0$

$\lim_{x \to 0^+} \ln y = \ln\left(\lim_{x \to 0^+} y\right) = 0$

$\lim_{x \to 0^+} y = e^0$

$\lim_{x \to 0^+} x^{\sqrt{x}} = \boxed{1}$