

Homework 15

b) $y = \sqrt{x-4}$
5 to 5.2

$$dy = f'(x) dx$$
$$dy = \frac{1}{2} (x-4)^{-\frac{1}{2}} dx$$
$$dy = \frac{1}{2} (5-4)^{-\frac{1}{2}} (0.2)$$
$$dy = \frac{1}{2} \cdot 1 \cdot 0.2$$
$$dy = 0.1$$

$$\Delta y = f(5.2) - f(5)$$

$$\Delta y = \sqrt{5.2-4} - \sqrt{5-4}$$
$$= \sqrt{1.2} - \sqrt{1}$$
$$= \sqrt{1.2} - 1$$

⑦ $f(x) = \sqrt{x+3}$ @ $a = 22$
 $L(x) = f(a) + f'(a)(x-a)$

$$f(22) = \sqrt{22+3} = \sqrt{25} = 5$$

$$f'(x) = \frac{1}{2} (x+3)^{-\frac{1}{2}}$$

$$f'(22) = \frac{1}{2} (22+3)^{-\frac{1}{2}}$$

$$= \frac{1}{2} (25)^{-\frac{1}{2}}$$

$$= \frac{1}{2} \cdot \frac{1}{5} = \frac{1}{10}$$

$$\underline{\underline{L(x) = 5 + \frac{1}{10}(x-22)}}$$

$$\sqrt{24.9} = \sqrt{x+3}$$

$$24.9 = x+3$$

$$x = 21.9$$

$$L(21.9) = 5 + \frac{1}{10}(21.9-22)$$

$$= 5 + \frac{1}{10}(-0.1)$$

$$= 5 - 0.01$$

$$= 4.99$$

$$\begin{aligned}\sqrt{25.01} &= \sqrt{x+3} \\ 25.01 &= x+3 \\ x &= 22.01\end{aligned}$$

$$\begin{aligned}L(22.01) &= 5 + \frac{1}{10}(22.01 - 22) \\ &= 5 + \frac{1}{10}(0.01) \\ &= 5 + 0.001 \\ &= 5.001\end{aligned}$$

$$\begin{aligned}8) \quad f(x) &= \sqrt{x-3} \quad @ \quad a=4 \\ L(x) &= f(a) + f'(a)(x-a)\end{aligned}$$

$$f(4) = \sqrt{4-3} = \sqrt{1} = 1$$

$$f'(x) = \frac{1}{2}(x-3)^{-\frac{1}{2}}$$

$$\begin{aligned}f'(4) &= \frac{1}{2}(4-3)^{-\frac{1}{2}} \\ &= \frac{1}{2}(1)^{-\frac{1}{2}} \\ &= \frac{1}{2}\end{aligned}$$

$$L(x) = 1 + \frac{1}{2}(x-4)$$

$$\begin{aligned}\sqrt{0.900} &= \sqrt{x-3} \\ 0.9 &= x-3 \\ x &= 3.9\end{aligned}$$

$$\begin{aligned}L(3.9) &= 1 + \frac{1}{2}(3.9-4) \\ &= 1 + \frac{1}{2}(-0.1) \\ &= 1 - 0.05 \\ &= 0.95\end{aligned}$$

$$\begin{aligned}\sqrt{1.01} &= \sqrt{x-3} \\ 1.01 &= x-3 \\ x &= 4.01\end{aligned}$$

$$\begin{aligned}L(4.01) &= 1 + \frac{1}{2}(4.01-4) \\ &= 1 + \frac{1}{2}(0.01) \\ &= 1 + 0.005 \\ &= 1.005\end{aligned}$$

Homework 16

$$5) f(x) = x^2 \sqrt{1-x^2}$$

$$f'(x) = x^2 \cdot \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) + \sqrt{1-x^2} \cdot 2x$$

$$= \frac{-x^3}{\sqrt{1-x^2}} + 2x\sqrt{1-x^2} \left(\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} \right)$$

$$= \frac{-x^3 + 2x(1-x^2)}{\sqrt{1-x^2}}$$

when $f'(x) = 0$

$$0 = -x^3 + 2x - 2x^3$$

$$0 = -3x^3 + 2x$$

$$0 = x(-3x^2 + 2)$$

$$\underline{x=0} \text{ or } -3x^2 + 2 = 0$$

$$3x^2 = 2$$

$$x^2 = \frac{2}{3}$$

$$\underline{x = \pm \sqrt{\frac{2}{3}}}$$

when $f'(x)$ is undefined

$$\sqrt{1-x^2} = 0$$

$$1-x^2 = 0$$

$$x^2 = 1$$

$$\underline{x = \pm 1}$$

$$8) f(x) = \frac{\ln x}{x} \quad [1, 3]$$

$$f'(x) = \frac{x \cdot \frac{1}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$$1 - \ln x = 0$$

$$\ln x = 1$$

$$x = e$$

$$f(1) = \frac{\ln 1}{1} = 0 \leftarrow \text{min}$$

$$f(e) = \frac{\ln e}{e} = \frac{1}{e} \leftarrow \text{max}$$

$$f(3) = \frac{\ln 3}{3}$$